SOAR Research Proposal – Summer 2013

Applying Network Analysis to Graphical Representations of Music

Faculty

- Nathan Shank, Associate Professor, Mathematics and Computer Science Department

Student

- Dylan Ricciardi, Mathematics Major

Time

- Ten week project starting May 28, 2013

Description of the project

A network is a relatively new mathematical idea which developed from graph theory. When we study networks we are studying the interaction of several objects amongst themselves. For example these objects could represent particles, people, cities, employees, vehicles, music notes, or many other real world concepts. Mathematicians model these networks as graphs with a vertex set representing the objects, and an edge set representing some type of interaction or relationship between the objects. We analyze the graphs to find important characteristics of their structure including characteristics of vertices and relationships between the vertices. After finding important characteristics of the graph, we then have to see what these characteristics mean in the context of the network. Thus we have a three stage process: (1) we model the network as a graph with vertices and edges, (2) we analyze the graph to find important characteristics, and (3) we see what these characteristics mean in the context of the problem.

We propose applying this process to study relationships in musical pieces. There are several ways that we can view music as graphs, for example we can use 12 vertices to represent the 12 notes and edges could represent note sequences. Thus we would have a directed graph where one vertex, v_1, is connected to another vertex, v_2, if v_2 is played directly after v_1 somewhere in the piece. As an example, Figure 1 shows the graphical representation of the note sequence for *Happy Birthday* where edges are colored according to how many times the note sequence appears in the song.

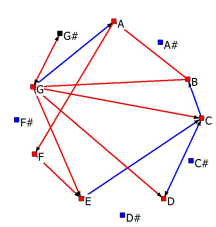

Figure 1: A graphical representation of Happy Birthday

There are other ways we could view music as a graph. We could consider the chords to be vertices and edges to represent chord sequences. This has been studied a little bit in previous literature. We would also like to consider pairs of notes to be the vertices (thus there would be 144

vertices) and edges could represent pair sequences. Thus an edge is present if a two note pair is followed by another two note pair.

Although some of these concepts have been previously considered especially related to music theory, we are intending to look at them from a more mathematical perspective. We are also interested in looking at graphs for music used in education, specifically at Moravian College. We'd like to see how the graph structure changes for basic music used in teaching fundamentals to upper level courses in music theory. We'd also like to analyze the graphs for music composed by faculty at Moravian College.

After we have a graphical representation of the network there are many different things we could consider. For example: (1) the number of 3-cycles (triangles) in a graph, (2) the number of n-cycles for different values of n, (3) degree characteristics (a degree of a vertex is the number of edges incident to a vertex), (4) diameter of a graph (the diameter is the furthest distance between any two vertices), (5) clique number of the graph (the largest number of vertices which are all connected), (6) independence number (the largest size of a set of vertices which are not adjacent to each other) etc.. Each of these would imply something different about the musical piece.

The main goal of this project is to have Dylan understand, experience, and succeed in mathematical research. Unlike many areas of mathematics, the study of networks allows undergraduate students to get into the forefront of research with minimal background reading, and ultimately to spend more time emersed in conducting research rather than reading background material. Since Dylan already has a vast knowledge of music (much more than me!) he should be able to quickly transform musical pieces into graphs and spend the majority of his time analyzing the structure of the graphs.

The study of graph structures is of particular interest to me because of my previous work in social network analysis which is an area of applied graph theory. I look forward to applying my knowledge of graph theory to another discipline to see what we can learn from a mathematical perspective. My last two SOAR projects (2008 and 2010), were both related to network reliability and social network analysis. Both of these projects resulted in papers co-authored with the students which have been accepted for publication in high quality peer-reviewed journals. I have the same expectations of the current SOAR proposal.

Roles and Responsibilities

The project will begin with some background reading on graph theory and mathematical theory of music. Dylan will be responsible for reading and outlining several articles, including survey articles and technical articles. This will serve as an introduction to the mathematical concepts used in analyzing networks. Dylan will also be working with specialized software (e.g. NODAL and UCINET) in order to analyze the data. He will also need time during the early stages of the project to convert the music into a form which we can input into the software (e.g. a relations matrix for

chords).

Timetable (10 weeks)

- Stage 0 (1 week) - Background readings, discussion
- Stage 1 (1 week) - Converting music into mathematical networks
- Stage 2 (2 weeks) - Analysis of the networks
- Stage 3 (4 weeks) - Interpreting the results of the network analysis in terms of what it means about the music. We would also include different pieces of music based on our preliminary findings. This would imply repeating Stage 1 - 3.
- Write up (2 weeks)

Summary of Benefits

This project will benefit Dylan by developing his understanding of mathematical research. This project was of particular interest to him because of his background in music. The project incorporates very different disciplines and Dylan will gain insight into interdisciplinary research.

Another goal of this project is to have the Dylan prepare a presentation suitable for an undergraduate conference. Specifically the Fall Eastern Pennsylvania and Delaware (EPaDel) section of the Mathematical Association of America (MAA) provides an excellent opportunity for Dylan to share his research with the larger mathematical community in the region. Dylan will present his work during one of the weekly math club meetings. This will benefit other students by making them aware of SOAR projects and show that math research is accessible to undergraduates. It will also benefit other students by showing them that mathematics is more then what is done in the class room - it is exciting, ever changing, and interdisciplinary.

This project will be a benefit to the department and for Moravian College since it will increase mathematical awareness on campus and enhance the scholarly atmosphere on campus. It will also serve to improve the level of undergraduate education for students. This interdisciplinary project would also increase the college, the department, and the students' awareness and appreciation of the ideals of a liberal arts education.

This project will give me a great opportunity to develop skills in mentoring undergraduates in mathematics research. Dylan is exceptionally bright and I am very excited to work with him on this project. I have very high expectations of for this project and know Dylan will work hard to get tangible results.

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Budget items ($100)

- Supplies: We are requesting fund to help cover the cost of the following materials
    1. UCINET Software - Used for network analysis - student version $40. Dr. Shank already has a personal copy on his computer however Dylan will need access to the software.
    2. NODAL Software - Used to visualize music as graphs (2 copies $30 each) $60.

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Faculty
• Nathan Shank, Associate Professor, Mathematics and Computer Science

Department Student
• Dylan Ricciardi, 3.31 Overall GPA, 3.00 Mathematics GPA, 17.50 Units Earned, Expected graduation Fall 2013

I will be requesting summer housing to assist with the project.

Rationale

In participating in this research project I hope to get a greater understanding of the applications of mathematical analysis inside interdisciplinary subjects. Music has always been a love of mine as has math, thus to analyze music through mathematical networking would give me a greater insight into my musical life while also exploring the fairly new world of network analysis and graph theory. Throughout the summer I hope to learn how to apply these strategies for analysis on any subject while focusing specifically on music. I was previously a music major at William Paterson University and transferred 4.75 units of music credits to Moravian. I completed music theory one and two, ear training one and two, intro to music education, electronic music, and many small and large ensembles. Though I am incredibly dedicated to the musical side of my life and have great conviction for the subject, I also understand that the business end of music is not very promising, especially in our economy. Since my second love is math, and I understand I can have a much more promising future for myself and my family, I decided to switch my major from music to math. This research project would give me a chance to work with both of my loves.

Expectations

Throughout the project I expect to delve into the format and structure of analysis and theories of math in interdisciplinary subjects such as music. From the start I expect to break down the music into a useable format, which would include placing notes into a chart or makeshift graph to first get the notes onto paper. This would also include whatever parameter we decide to analyze; such as the chord structure the notes are a part of or the sequential order in which the notes occur. This will be repeated for different types of music, including different keys, genres, and time periods.

After breaking down the music into a useable form I will use UCINET and other networking programs to help analyze the music. In a given piece of music there are chord structures and the sequence of notes generally outline a certain chord in a given key. Through networking analysis and graphical analysis the behavior of music can be monitored. Such questions can be asked as whether or not the music follows the general form and chordal progressions that are most common, how far from the norm they stray, and if non-chord-tones follow chord tones are they scalar, chromatic, or neither. This is to say if a note is not within the given chord for the particular section of music being analyzed, are those notes within the scale the current piece is working around, are they within a half step (one 12 tone value) of the preceding note, or is the relation between the sequential notes something different completely.

After seeing the patterns of notes and chords within the music, I can then draw conclusions on whether or not the composer was traditional, that they used the progressions found most often in their time period or within their genres, or whether they were one of the firsts to stray from the norm and venture into new territory. I also expect to have a much better grasp on how to analyze anything through mathematics, specifically through networks and graph theory. By the end of the project I will be better equipped to tackle more analytical problems and assess certain aspects of the discipline in which I'm studying.

Finally once the project is complete, I hope to be able to present it to other via Scholar's day, possible epsilon talks for our math department, or even present my findings at a regional meeting. This research project would not only give me the chance to work with both math and music in one, but also give me important skills by using different software and learning to type technical mathematics in LaTex. This project could not only benefit myself, but also benefit the Moravian mathematics community through talks and presentations, and by showing that math is truly used in ever facet of life, even if it does not present itself as a useful application at first sight; many may thing music could not be evaluated using mathematics, however I want to prove that this is very possible!