SOAR Research Proposal – Summer 2019
Diameter Restricted Polyominoes and Optimal Network Reliability

Faculty: Nathan Shank, Associate Professor, Mathematics and Computer Science Department
Dates: June 3, 2019 - August 9, 2019 (10 weeks)
Students: Alvaro Belmonte, Junior, Mathematics Major, Graduate in Spring 2020
Rey Anaya, Sophomore, Mathematics Major, Graduate in Spring 2021.
Title: Diameter Restricted Polyominoes and Optimal Network Reliability

The Moravian College Mathematics Program recently received an *National Science Foundation Research Experience for Undergraduate (NSF-REU) Site* grant. The three year grant provides summer funding for approximately 15 extremely talented undergraduate students from across the country to come to Moravian for a 9 week research experience in mathematics concurrent with SOAR. The NSF grant is intended to support students outside the home institution, however, if a qualified Moravian student can internal funding they can participate in the program.

Participating in the program has many benefits to Moravian and its students. Participating in such a program is a life-changing activity for students, Moravian students would have the opportunity to work with some of the brightest students from across the country, life-long associations develop in these programs, and students usually get at least one peer-reviewed publication. The program is centered around computational and experimental mathematics (CEM) which harnesses the power of computers to help further the boundary of mathematics. The re-emergence of the field has lead to some stimulating new research and we want to prepare our students to be at the forefront of these developments.

The program engages in discipline-appropriate scholarly research by using a cohort based model where each student is involved in two projects with 3-5 students and a faculty mentor. If funded, the SOAR students will be in at least one of the projects below with Dr. Shank as their mentor. Each cohort is required to give weekly presentations to the entire REU community which adds another layer of accountability, allows students to understand many different projects, and gives the students practice presenting their work. The SOAR students will satisfy all of the requirements of SOAR including working with a Moravian faculty, making a presentation to the SOAR community, working full time on their project, working with public relations, making a Scholars Day presentation, and writing a final report.

Description of the Project

Diameter Restricted Polyominoes: A polyomino is a two dimensional geometric figure made up of square cells that are joined along an edge. We classify the size of the polyomino by the number of cells, n. The classic game of Tetris is played with polyominoes of size $n = 4$. Investigations into polyominoes generally fall into two different categories: tiling and counting. Tiling problems involve finding ways to tile regions with polyominoes and counting problems involve counting the number of possible polyomino configurations based on the size n and other restrictions.

The Online Encyclopedia of Integer Sequences (OEIS) is a database of integer sequences containing over 320,000 sequences and over 700 sequence related to polyominoes including all polyominoes (OEIS A002212), free polyominoes (OEIS A000105), free polyominoes with holes (OEIS A001419), and fixed polyominoes (OEIS A001168). For small values of n, counting polyominoes can be done by hand, however, as n increases, the number of polyominoes increases to where hand computations are difficult if not impossible. For example, when $n = 20$, there are over **630 billion** polyominoes.

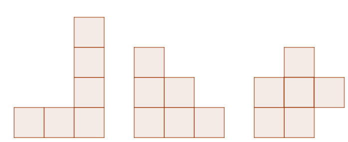

Figure 1: *Examples of a polyominoes with $n = 6$ with diameters 5, 4, and 3.*

In this project we want to count polyominoes of size n with a fixed diameter d. Here diameter is measures as the maximum number of squares traversed to get from one square to another. For example, if we are allowed to use $n = 20$ cells, how many polyominoes are possible if we restrict the diameter to $d = 10$? Finding the relationship between n and d would allow us to characterize polyominoes by diameter. When n is small, we can do this by hand, but as n increases, we will apply computational and experimental mathematics to help generate results and formulate conjectures.

Optimal Network Reliability: In mathematics we model a network as a set of vertices and a set of edges between the vertices. This allows us to model many different applications with one mathematical structure. For example, computers and connections, power stations and power lines, people and friendships, cities and roads, atoms and bonds, regions and animal migration paths, and words and semantics can all be modeled as a mathematical network. Network reliability considers how fragile or robust a network is under certain conditions. Power companies need to understand weaknesses in their power grid, health professionals need to understand the spread of disease and the impact of vaccines on a community, data centers need to keep their network operational and their data safe.

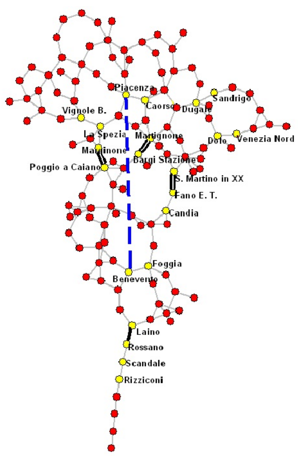

Figure 2: Italian 380kV power grid & critical edges.[1]

When considering network reliability there are two things we need to consider: what make a network operational and what is prone to failure (vertices, edges, or both). A recent network reliability measure called the *component order edge connectivity COEC* parameter[2] considers edge failures and assumes a network is operational if there is a component of order k or more. For example, GPS must have at least three satellites *connected* in order to accurately determine a location, therefore $k = 3$ in this case. As long as k vertices are connected the network is operational. The *COEC* measures the least number of edges that can fail in order to make the network fail. In this project we will attempt to find the maximum and minimum values of *COEC* for networks with a fixed number of edges and vertices. This project will continue work started by by a previous SOAR student, Phillip Gaudreau ('08), which found minimum values for $k = 2$ and $k = 3$.[3] The results for larger values of k are still unknown and will be explored in this SOAR project.

Roles and Responsibilities

Alvaro and Rey will be involved in all aspects of the project. The literature review will begin as soon as the spring semester is complete. This will involve reading several assigned articles as well as finding other articles and sharing them. This will serve as an introduction to the mathematical concepts used in the projects so they can begin working as soon as SOAR start.

After the literature review, we will spend time exploring both projects. We will begin by examining small cases which can be easily computed by hand. For example, when $n = 4$, there are only 36 polyominoes and when $n = 5$, there are 137 polyominoes. We will then need to develop a computer program which can efficiently enumerate the polyominoes and their diameter. Both students have at least two courses in computer science which provide sufficient background for developing a program. Based on the results, we

[1] P. Crucitti, V. Latora, W. Ebeling and B. Spagnolo, *Locating Critical Lines in High-Voltage Electric Power Grids*, Fluctuation and Noise Letters, **05** (June 2005).

[2] F. Boesch, D. Gross, W. Kazmierczak, C. Suffel and A. Suhartomok, *Component Order Edge Connectivity—An Introduction*, Proceedings of the Thirty-Seventh Southeastern International Conference on Combinatorics, Graph Theory and Computing, Congo. Number, **178** (2006), 7-14.

[3] P. Gaudreau and N. Shank, *Component Order Edge Connectivity for Graphs of Fixed Size and Order*, Ars Combin., **116** (2014), 225-233.

will develop conjectures for larger, computational difficult, values of n. Ultimately we would like to be able to prove the conjecture for all values of n, however, at a minimum we will be able to contribute several new sequences to the OEIS.

TimeTable

Week	Objective
Week 1	Finish background reading and create presentations including summary of background reading and timeline.
Week 2-4	Develop and write a computer program to help enumerate possibilities. This will involve understanding an efficient way to write the program. Testing will need to be done to verify the validity of the results. Continue to refine the program and have the program run larger values of n. This may involve using the Computer Science cluster available on campus.
Week 5-6	Analyze results to start to develop conjectures based on the output. Search the OEIS for our new sequences to see if they have been discovered in other areas of mathematics.
Week 6-9	Develop ways to formally prove our results for all values of n. We will also continue to refine the program and run the program for larger values of n.
Week 10	Begin to prepare results for publication and develop a writing plan for the future.

Why Two Students?

These projects are substantial in their scope and complexity and will require a team to solve as indicated by the fact that my previous SOAR student, who went on to graduate school in mathematics, was only able to complete $k = 2$ and $k = 3$ for one of these projects. Problems in mathematics have become increasing more difficult and, therefore, it is no longer done in isolation but requires a collaborative effort. Having lead several REU cohort groups over the past three summers, Dr. Shank understands the academic ability needed to participate in this high level research environment expected by the NSF. Alvaro and Rey are two of our most academically talented students who have shown the potential to conduct research at a high level. They should have the opportunity to work on these projects and engage in research with other extremely talented students. Alvaro and Rey have both indicated their interest in going to graduate school in mathematics and participating in this type of research, having a peer-reviewed publication, and giving research presentations will help them gain experience and have a significant impact on their applications.

Engagement in Discipline-Appropriate Scholarly Research

The projects will follow the general outline of mathematical research. First we do a literature review to understand the problem and learn problem specific techniques and tools. Next, we find empirical data to help us develop conjectures. This will be enhanced by the use of computational mathematics to help us understand large values of n which are not possible to calculate by hand. The empirical data leads to conjectures and then we try to prove the conjecture or revise the problem. Both students will have a major role in all phases of the project. At the end of the summer the cohort will decide how they can start to prepare their results for publication. The students will also gain experience communicating, both written and verbally, results to different audiences. This is a vital stage of the research process and will continue after the program has concluded.

Goals, Contribution to the Discipline, and Opportunities to Share Work

This project will prepare the students to become the next generation of mathematical scientist efficient in using computer-assisted techniques to investigate mathematical patterns and properties which pushes the boundary of mathematical knowledge. We expect at least one peer-reviewed journal paper and several presentations as a result of each of the SOAR / REU project. The students will share their work at the regional *Eastern Pennsylvania and Delaware* section meeting of the *Mathematical Association of America*, Moravian Scholars Day, and the annual *Moravian Student Mathematics Conference* in February.

SOAR Student Statement - Summer 2019
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Faculty: Nathan Shank, Associate Professor, Mathematics and Computer Science Department
Dates: June 3, 2019 - August 9, 2019 (10 weeks)
Student: Alvaro Belmonte, Mathematics Major
Graduation: Spring 2020
Housing: Yes

Title: Diameter Restricted Polyominoes and Optimal Network Reliability

Rationale

"If you are the smartest person in the room, then you are in the wrong room." Attributed to no one, this quote has guided my intellectual ventures since I entered higher education. I have always tried to challenge myself by taking hard classes, trying to expand my abilities, and doing independent learning outside class. The main reason for doing this SOAR Project is to challenge myself and sharpen my intellect and mathematical tools that will help me in the future in graduate studies. This will challenge my knowledge that I have acquired during the last years of higher education, not only by directly applying what I have learned in classes, but by expanding the critical thinking skills developed during those years.

Participating in SOAR will help me develop, understand, and experience the process of mathematical research, where we try to find the answer to a difficult question. Not necessarily directly, but by finding back alleyways through simplifying the question, finding patterns, and trying to generalize results.

This program will not only give me experience in research, but it will give me more insight on which branch of mathematics I want to specialize in for my graduate studies. I am drawn to study Analysis and Algebra because I like the abstractions involved in analysis and algebra combined with the way we make arguments using simple definitions to show complex results. The results build upon themselves to give us a more complete picture.

I am interested in the studies of experimental and computational mathematics because, with my background in engineering when trying to answer a question, I try to gather empirical data to try to see patterns. This is exactly what computational and experimental mathematics involves. We can see that computational mathematics is becoming more useful due to the growth of computational power, so much so that now large problems instead of taking years to try to compute, take days, and therefore they are realizable. Also, the fact that in mathematics the discoveries of one branch tend to lead to massive discoveries in a lot of other disciplines means that what we do in our research could be used to further other areas.

Last, but not least, I think that working in a group on difficult and unsolved problems will be fun and rewarding. During this spring semester, I am leading a group of freshmen engage in a research project which has allowed us to understand mathematical research. I enjoyed the collaborative work environment within my group and I would like to have another experience like that, but this time with more a more substantial project. In this same class, I also enjoyed the effort my group has made trying to get a result of an unsolved problem and to understand why that solution actually works. The SOAR experience would not only enrich my summer intellectually, but also in the fall I will begin a year long Honors project and my experience in SOAR will help develop my research skills.

I am interested on both projects, so I am excited to work on both of them. I am interested in the Optimal Network Reliability project because when I was studying engineering, I was, and still am, really interested on power distribution and networks. Hence, I would learn about a branch of mathematics that would be easily applicable for such a subject. I enjoy when mathematics has easy to explain applications, yet the problems can be very difficult to solve. I am also interested on the project of Diameter Restricted Polyominoes because it would help me work on a part of mathematics which is not my forte, therefore strengthening my skills for my future academic career. This problem would require me to use counting skills to be able to enumerate all the possibilities.

I have applied to other REU programs and have been waitlisted for one. Having transferred from Northampton Community College, this is my only summer to gain valuable research experience and I am excited about the opportunity to do mathematical research.

Expected Outcomes

The SOAR/REU will let me work with other talented students and faculty. This will give me valuable experience working on a team project, which is an important skill, not only for graduate school, but for my career. It will give me experience that will help prepare me for graduate school and doing research It will let me expand my abilities to communicate my ideas and work to math and non-math audience, helping me with my oratory and public communications skills. I will let have a first-hands experience on how to write a mathematical paper, and hopefully the process of submitting it to a mathematical journal. The SOAR/REU is not only developing my personal skill, but it is also letting me work with my contemporary mathematical peers and let me have national connections among them, that hopefully last during my academic career.

At the end of the project I hope to have a draft of a paper, which I would be excited to submit for publication in a journal. As I said before, it will help me with my oratory and public communications skills because I will be doing several presentations to peers in regional and local conferences. It will let connect with students and faculty which would let me form a network with peers with similar academic interests. The SOAR/REU will help me select and have a better understanding of the field of mathematics I want to keep studying in graduate school.

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Faculty: Nathan Shank, Associate Professor, Mathematics and Computer Science Department
Dates: June 3, 2019 - August 9, 2019 (10 weeks)
Student: Rey Anaya, Mathematics Major & Computer Science Minor
Graduation: Spring 2021
Housing: Yes

Title: Diameter Restricted Polyominoes and Optimal Network Reliability

Rationale

Throughout high school and college, my favorite subject has been mathematics. Mathematics has always been interesting to me because of the precision involved, however, it is also one of the more difficult subjects for me to learn. The reason why I have been successful is due to the fact that I am dedicated and enjoy the time and effort needed to understand the depth of the subject. In my high school mathematic courses, it was just the regular "here is a topic, go do exercises 1-20 on page 50 from the textbook." But this type of work is tedious and repetitive and I never fully understood the concepts. But once I got to college we broke away from this idea of doing repetitive exercises and moved towards solving more complex problems in order to understand why. This is where I struggled to transition myself from my old habits. This did not stop me from loving mathematics, but instead, strengthened my interest in the subject.

Before coming to Moravian, I never once thought that you could *prove* something in mathematics, or how to even start. This concept was foreign to me. I first got exposed to writing proofs in my discrete mathematics course (MATH 212) in which we started asking questions such as "why is this true?" rather than "what is the answer?" This is by far one of my favorite things about mathematics, and that is where SOAR will help me gain experience of doing research and give me the opportunity to work with other talented students and faculty.

I am currently enrolled in MATH 298.2, Introduction to Mathematical Research. One goal of the course is to solve an unsolved problem. Of course, the odds of solving the original problem are low, but that is not the main point of this course. The idea is to learn the research process for upper level mathematics and gain experience in conducting research. This means being able to understand previous work, analyze the problem, develop conjectures, try to prove those conjectures, and, if needed, tweek the problem. So far this course is one of my favorite courses because this is the first time I have been exposed to the mathematical research process. We chose a problem that has never been solved before by anyone. We then modified the problem to make it more interesting, and being able to solve a problem that no one else has solved before is mindblowing. If selected for SOAR, I will be able to gain more experience in conducting research in mathematics.

Moravian is having an REU and if accepted into SOAR, I will be able to work with other extremely talented undergraduate students who have similar goals. This will not only be an amazing experience, but it will also help prepare me for next year when I plan to apply for other REU's across the country. This is a very important step for me since my goal after Moravian College is to go to graduate school in mathematics and eventually become a mathematics professor. Although mathematics is thought to be done in isolation, in reality, people collaborate with peers often. Working in groups is encouraged, allows brainstorming of ideas, and allows us to further our research more efficiently. I am excited to build relationships with other REU students and faculty which I plan to continue after the summer.

Not only will SOAR help me gain research experience, participating in the REU will help me learn the skills needed to share and publish the results. The network reliability problem is particularly interesting to me because it has some very real and important applications. I have studied some graph theory topics in MATH 212, Discrete Mathematics and Introduction to Proofs, and the problem which I am working on in the research class is related to graph theory and networks. I also find it interesting how mathematics has evolved to the point where we are now using programs to help with computational mathematics. This allows us to work on problems that would have been impossible to do by hand just a short time ago. The Diameter Restricted Polyominoes problem will allow us to explore more into how we can use computer programing to help us find solutions and patterns. Both of the projects outlines are topics which are not covered in normal courses and it would be fun to work on these problems.

I know SOAR will not be easy and will be a challenge, but that is exactly what I want. I want to struggle as much as I can and get the most experience out of my four years at Moravian as I possibly can. In order to learn how to conduct research, being told how to do it is **not** effective. The best way to learn the research process is to actually be part of a team of students who work together to solve a problem, which is what I will be doing.

Expected Outcomes

After I have completed my SOAR project, my main take away will be to have gained experience in how to conduct mathematical research. I hope to learn as much as I can about the research process. I also want the experience of working with fellow undergraduate students who are in the same field as me. Not many Moravian students are interested in becoming mathematicians or mathematics professors, but since I will be working with other students with similar interests, this can help me to better my skills and exchange ideas about mathematics with them. Although proving something is important, I am more excited about engaging in the research process of how to solve something.

At the end of the summer, I hope to begin writing a research paper for publication. I plan to present our findings at local and regional conferences. This will also help encourage other mathematicians and maybe someone will be interested in our research and build upon it. I also intend to continue working on the problem as a possible independent study project or in another research class.