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CHALLENGING STUDENTS WITH LEARNING DISABILITIES: CREATING DIALOGUE ABOUT MATHEMATICAL WORD PROBLEMS USING A SCHEMA-BASED STRATEGY

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ABSTRACT

This research study used qualitative methods to examine the learning processes of four fourth-grade students with learning disabilities as they engaged in dialogic problem solving of addition and subtraction word problems. The students learned specific problem solving steps through direct instruction in how to use Representational Schema Diagrams, developed by Jitendra and Hoff (1996). To practice the strategy, the students engaged in conversation and explanation in teacher directed small groups and in student paired groupings. They used the strategy steps and the schema diagrams to guide their dialogic process as they solved "rich" word problems. The researcher documents both improvements in the students' process and communication skills and points of confusion the students encountered.

ACKNOWLEDGEMENTS

I would like to thank the students who participated in my study. They read and solved a lot of word problems this year! I appreciated their enthusiasm and excitement about being involved in a college project and their willingness to try the new strategy that we learned.

I would also like to thank my colleagues for their support of my action research project. When we devised the schedules at the beginning of the year, they helped me find mostly uninterrupted time of day for math instruction. As a support teacher, I have been in many classrooms and worked with many of the teachers in my building. I learn from them how regularly achieving students respond to problem solving situations, and I see how they carefully guide students in the process of understanding and solving word problems.

I want to thank my teacher researcher class. I feel a bond with all the teachers in our class. Everyone has shared a lot of knowledge, and even wisdom, as we discussed everyday issues of classroom teaching and the big ideas of philosophy that underlie our everyday decision making. I will miss seeing these colleagues when classes end.

Thank you to our professors at Moravian College who continue to refine this program. I had never heard of teacher action research before this experience. It has been liberating to envision myself as a researcher as well as an instructor. I hope teacher research continues to gain respect among educational decision makers.

My thanks to my thesis committee for helping me to make my document the best it could be. I appreciate the time and thought they invested in my work, and the improvements to the final document their suggestions enabled.

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RESEARCHER STANCE

I believe that students need to learn to think mathematically. Problem solving is the practical part of math. In our daily lives we are rarely presented with a page of computation problems to solve, but often are confronted with situations where our knowledge of math reasoning is required.

Thinking mathematically has never come naturally to me. I am not an eager problem solver. When faced with a mathematical problem, like how many cans of paint to buy to paint a room, I prefer to let someone else figure it out. Some of my students also seem to be afraid to think through problem situations. They seem to guess, usually choosing to add the numbers in the problem. Where does this fear of problem solving come from?

In elementary school, I was not good at math. I struggled to memorize the addition and subtraction facts. I still do not automatically know the answer to 7 + 4! I never got better at school math. I am sure that I have the ability to understand it, but something about the way mathematics was presented to me in school paralyzed my brain. I don't want my students to think math is paralyzing or too hard. Some facts and procedures must be practiced, but thinking needs to be practiced, too. I wondered how to structure problem situations so students could be successful at creating solutions.

A person is more likely to persevere at a difficult task if he or she is interested in it. I noticed that my students were most excited about math class when they played games or used math tools. I thought about studying the use of calculators for solving word problems. It would be motivating and would support their weak calculation skills, allowing them to focus on the math reasoning skills. But when I observed students working with calculators, it drew more attention to the calculation phase. Eagerly they typed in the numbers and added. This approach seemed too simplistic. I would need to do more than support calculation skills. I decided to change my focus to reflect the complexity of problem solving.

I wondered how to get students to slow down and plan how to solve a word problem. I started to pay attention to what happened when my students solved two word problems each day. They learned to underline the question and bracket the number facts in the story. Then they had to decide what to do. Sometimes they used key words like "in all" or "are left" to decide correctly. The students I supported in regular math would add if the paper was about adding, or multiply if it was about multiplication. They did not want to read the problem, just pick out the numbers and do the operation. It occurred to me that the problems were too predictable or too simple to require thinking. Would more challenging problems stimulate thinking or be too difficult and result in guessing?

I wanted the students to be actively engaged in the problem solving tasks. When we solved problems together, some would be involved and others would wait to hear the solution and then copy it down. I noticed this lack of attention with my included math students and with the math students I taught in the resource room setting. The only thinking they did was to be alert for the answer. Problem solving conversation was already a part of class. I planned to structure the conversation carefully, so students could use that structure independently.

Talking with a partner would allow students to engage in more talking and problem solving. I examined my experience with partner work. Partner work in my math class usually involved studying math facts with flashcards. Sometimes the partner work was problematic because the students were unkind to each other. They eagerly pointed out their partner's mistakes and ignored the correct answers. I wondered if they could learn to work together more effectively. Would talking about math problems stimulate them to think mathematically?

I decided to ask this question for my research project:

What will happen when I challenge students with learning disabilities to think mathematically to solve a variety of word problems, and to engage in discourse with their peers as they create solutions together?

REVIEW OF THE LITERATURE

In 1989, the National Council of Teachers of Mathematics called for a change in math instruction away from computation and toward higher-level math reasoning and problem solving. The council held that *all* students should become capable and resourceful problem solvers and learn to reason and communicate mathematically. These ideas are based on constructivist theory. Constructivist approaches focus on deep levels of understanding. The teacher's role is to guide students as they interact with materials, ideas, and other people. Through discussion and exploration, students create their own knowledge by discovering concepts. Students are expected to assimilate new concepts into prior knowledge. I was not convinced that this approach would work for me or for my students with learning disabilities. Then, with the passage of No Child Left Behind legislation, I learned that all students would be assessed on the same measures and held to the same standards. They needed to practice the kinds of thinking that their classmates in regular education classes learned. Even though I felt that to expect all children to be proficient at the same level and at the same time denied that learning disabilities existed, I began to search for the support students could use to approach proficiency in higher level thinking tasks. I had to consider all approaches.

Thornton, Langrell, and Jones (1997) believe that students with learning disabilities do benefit from programs with a constructivist framework. They advocate a broad and balanced math curriculum instead of one unnecessarily focused on computation. Broad curriculums emphasize number sense and estimation, data analysis, spatial sense and geometric thinking, patterns and relationships that lead to algebraic thinking, and the supportive use of technology. These researchers assert that these different kinds of mathematical thinking encourage the development of math skills, including numerical reasoning, and enable students with learning disabilities to use mathematics more productively..

Though these domains can be taught in different ways, an advantage of constructivist models is that different approaches are valued, allowing more children to succeed. Thornton, Langrell, and Jones (1997) cite four case studies from other researchers as examples of learning disabled students finding success using constructivist methods. Behrend (1994) cited in Thornton, Langrell, & Jones (1997) suggests that teachers should observe and listen to children's thinking patterns. In Behrend's case study, a student, "Dan", was not successful in applying rules or steps to solve word problems, but did succeed in devising his own way to solve a non-routine math problem when allowed to create his own solution. Behrend suggests that teachers should not always be proactive in teaching problem solving. She concluded, "Instruction should build on children's current understandings and promote the development of increasingly more efficient problem solving strategies" (p. 147).

Constructivism respects students' abilities and centers instruction around the student, not on the teacher. Delpit (2002) writes that "teachers must not only see their students as non-deficient, they must understand their brilliance" (p. 42). I wanted students to have a chance to be brilliant in math class. By giving them the opportunity to use models, conversation, and creativity to figure out math puzzles weekly, perhaps they would find their own smart solutions. And I would be there to praise them. Freire (2002) says, "Fight at their side," supporting both action and reflection. To me, this means that as the children work to understand math, their teachers must work to understand the children (p. 39). But Dewey (1938) cautions that, "the belief that all education comes about through experience does not mean that all experiences are educative" (p. 25). I was willing to try a constructivist approach, but I wanted to structure the experience so that learning was likely to happen. My next step was to review the literature about effective instructional strategies.

In this list, Vaughn and Linan-Thomson (2003) summarize effective instructional strategies for students with learning disabilities that have been validated by research:

1. Teaching in a small group.
2. Modeling and teaching self-questioning and self-checking strategies (metacognition).
3. Providing direct and explicit instruction.

4. Engaging in higher-order processing skills and problem solving to integrate knowledge and skills.
5. Learning when, where, and how to apply strategies.
6. Using ongoing progress monitoring of specific skills in academic areas to guide interventions.
7. Controlling task difficulty.
8. Teaching the building blocks of reading and writing (phonemic awareness and writing speed).

Torgeson (1996) specifies that these strategies need to be taught in a way that is more explicit, more intensive, and more supportive than similar techniques in regular education. Still, Vaughn and Linan-Thompson (2003) acknowledge that some students do not respond to interventions that have been effective for students with similar and related learning problems. In short, there is no prescriptive solution for teaching students effectively. Individual differences require individual solutions. It is important for teachers to search for all the factors that interfere with learning and devise individual interventions for struggling students.

Neurological and emotional factors influence a student's ability to learn. Students with learning disabilities often have a history of academic failure, which leads them to believe that they have no control over their learning. Students may find it painful to learn because of neurological problems with memory, attention, perception, coordination, and handwriting. Students who have learning disabilities are often overwhelmed, disorganized, and frustrated in learning situations. A chart from *Teaching the Tiger* by Dornbush and Pruitt (2002) illustrates how different neurological disorders affect student academic functioning (see Appendix A).

Vaidya (1999) refers to attribution theory as the theoretical basis for metacognition in a paper discussing the importance of metacognitve learning strategies for students with learning disabilities. Attribution theory says that our opinion of ourselves determines how we behave. Those who believe that they can't learn have low motivation for learning tasks. They attribute their success to luck. I have heard students say, "When I study, I do worse. So I never study." In this way, "Attribution influences learner motivation" (Fulk, 1996, in Vaidya, 1999, p. 3).

It is vital that students with learning disabilities gain a metacognitive awareness of their own learning needs. Then they need to learn how to use strategies and to choose those that are effective for them, thus giving them control of their learning. Students must connect their efforts to their academic successes, so that they are willing to strive. Students with learning disabilities can be taught to know themselves as learners – explicitly, intensively, and supportively.

I began to search for effective strategies for teaching math to students with learning disabilities. I wanted detailed descriptions of cognitive and metacognitive steps in the problem solving process. I found that researchers used careful analyses of students' errors in the problem solving process to guide the design of effective strategies. They used these error analyses to create step-by-step procedures for students to follow when they attack word problems

Hohn and Frey (2002) explain that the process of understanding and solving word problems proceeds through four sequential phases:

Stage 1– Problem Representation: a) translate (retell)

b) integrate (relate to knowledge)

Stage 2 – Solution Planning: select a procedure or operation (<, +, -, etc.)

Stage 3 – Solution Execution: compute

Stage 4 – Solution Monitoring: detect errors or verify sensible answers

They developed a training strategy with the acronym SOLVED for regular education students in third, fourth, and fifth grades. First, the students memorized the steps. Then their regular math teacher modeled the process as the teacher solved a word problem. Following the demonstration lesson, pairs of students solved two similar problems and explained how they used SOLVED in one of the problems. The results indicated that students in elementary school could be taught to use a simple meta-cognitive strategy. Use of the strategy resulted in a superior learning rate for the third and fifth grade groups. Some issues were indicated by the research findings. The researchers noted that students most often skipped the representation and planning phases of the problem-solving process. Most of their thinking was centered on the computation phase. Hohn and Frey (2002) recommended that "increased emphasis on representational skills will need to be included in teachers' instructional methods" to overcome students' tendencies to jump to the solution phase (p. 6).

Montague, Warger, and Morgan (2000) reported that students with learning disabilities are often unable to represent word problems in mathematical language. Their strategy, named *Solve It!* was developed for high school students with learning disabilities. In their study, most students' problem solving scores improved to about 70% accuracy after they learned to use the seven-step checklist. The 70% goal is based on the average score for regular education students. The authors considered students with learning disabilities successful if they could achieve average performance, because this was considered successful for regular education students. Students who learned the *Solve It!* strategy attacked complex word problems with perseverance, but not with success. When Montague (2001) analyzed the errors by students using the *Solve It!* strategy, she found that 75% of the errors were process or operation errors. Of those thirty processing errors, twenty-eight were errors in setting up the math sentence to solve the problem.

Clearly, students needed support at the initial problem representation phase of solving word problems. Jitendra and Hoff (1996) created a Graphic Representational Strategy for math word problem solving. This strategy helps learners create a visual representation of word problems in a way that leads to a solution. Their approach focuses on analyzing the semantic structure of word problems. The important words and numbers are highlighted in the sentences, and then this key information is mapped into the chosen representational schema diagram. The schema diagram sets up the math sentence to be solved. Three schema diagrams, Change, Group, and Compare, represent all one-step addition and subtraction problems. Two other schema diagrams represent multiplication and division problems. Combinations of these diagrams can be used to solve multi-step problems. The Graphic Representational Strategy uses the schema diagrams to directly address students' weak performance in the problem representation phase. The strategy combines the schema diagrams with a strategy checklist, known by the acronym FOPS. FOPS stands for the four steps of the strategy:

Find the problem type

Organize the facts into the schema diagram

Plan what operation to use

Solve by performing the calculation.

These steps are expanded with specific metacognitive guiding questions for each problem type to guide and support students' thinking as they solve math word problems (see Appendixes B, C, and D).

Early in my research, I attended a training seminar on how to implement this Graphic Representational Strategy approach to teach mathematical word problem solving. Dr. Jitendra demonstrated how to teach this representational schema diagram strategy. She provided lesson scripts for each type of schema diagram, metacognitive checklists for each problem type, samples of problem solving probes, and schema diagrams for each problem type.

I decided to try the diagrams and the instructional strategy with my students with learning disabilities because the method was researched based, incorporating important elements of effective math instruction for students with learning disabilities.

The following section examines the Graphic Representational Strategy in reference to descriptions of effective mathematics instruction for students with learning disabilities, synthesized from the work of many math and special education researchers. Effective strategies accommodate the special learning needs of students with learning disabilities such as: memory impairment, difficulty generating or understanding strategies, difficulty sequencing, and difficulty labeling, storing, and retrieving information in memory (Dornbush & Pruitt, 2002).

One effective strategy for students with learning disabilities is the use of scaffolded instruction. The aim of scaffolding is to create independent self-regulated learners. Initially, teachers provide support and structure; then systematically remove the guidance as student competence increases (Ellis & Worthington, 1994). Rosenshine and Meister (1992) describe scaffolds as an intervention at a middle level, between the specificity of behavioral objectives that lead to skills and the lack of instruction that many criticized in discovery learning settings. Scaffolding is a socially mediated, dialogic form of learning. "As the teacher models and verbally elaborates each step as the process is performed, the child gradually attains competency." (Ellis & Worthington, 1994, p. 31)

The schema diagrams are important scaffolding tools because they "allow students with poor memory abilities to organize information using semantic relations and thus to acquire adequate word-problem solving skills" (Jitendra & Hoff, 1996, p. 8). The graphic representations or schema diagrams are visual scaffolds designed to show students the relationships between numbers in a word problem. The schema instructional strategy is a dialogic method of modeling, discussing, and visually supporting the learning with the diagrams and the FOPS checklist.

Carnine (1997) describes effective instruction in math for students with learning disabilities. He discusses five design principles:

1. Abandon low-priority objectives and focus on big ideas.
2. Teach conspicuous strategies.
3. Use time efficiently.

4. Clearly communicate strategies in an explicit manner.

5. Provide appropriate practice and review to facilitate retention.

He asserts that building math instruction based on these principles "should enable students with learning disabilities both to acquire basic skills and to solve challenging problems in mathematics" (p. 139).

Carnine's (1997) first design principle for math instruction is to organize content around "big ideas". He refers to Woodward's (1994) investigation, which showed that using a *big idea* in many different science situations improved scientific problem solving. *Big ideas* in math are important concepts with connections to many problem situations. When students understand a *big idea* and its connections, fewer formulas need to be memorized.

In Jitendra and Hoff's (1996) schema diagram approach, the diagrams and the metacognitive strategy questions are connected to important *big ideas* in math. Each diagram forms an equation, illustrating the concept of equivalence on both sides of the equation. The concept of the total is important for students to perceive in order to map the information into the diagrams correctly. The term *big number* is used to indicate where the total is represented in the problem. The diagrams also support the concept of inverse operations. When a fact is missing, a question mark is inserted. If the math sentence that is created has a missing number, the students learn to work backward by doing the inverse operation.

The second principle is to teach conspicuous strategies. One challenge of instruction for students with learning disabilities is to develop strategies that are "just right". McDaniel and Schlager (1990, as cited in Carnine, 1997) explain that strategies can be so specific that they apply to only specific situations. At the other extreme, strategies can be so general that they do not reliably lead students to solutions. One such unreliable strategy is the "draw a picture" strategy. Unless the picture drawing is explicitly taught to reliably represent the problem, many errors can occur.

The schema diagrams developed by Jitendra and Hoff provide a reliable picture for problem solving. The diagram and the metacognitive strategy checklists combine to create a conspicuous strategy that students can follow to label, store, and retrieve the information they need to understand problem situations. The problem solving steps (FOPS) are the same in each schema checklist. The diagrams and checklists give structure and support for complex thinking. They do not simplify the mathematical work. Students can be challenged and supported at the same time.

The third principle of using time efficiently addresses the perennial problem in teaching students with learning disabilities - catching them up with their peers as efficiently as possible without overwhelming them. When thinking of instruction in terms of *what* and *how* to teach, an efficient approach to choosing *what* to teach should be guided by abandoning low priority objectives and focusing on the big ideas in the curriculum. To decide *how* to teach, remember that an efficient strategy adheres to the Law of Parsimony, meaning it "explains the most in the simplest way" (Carnine, 1997, p. 4).

The graphic representational strategy is efficient. In Jitendra and Hoff's approach, students need to learn just three schema diagrams and one set of metacognitive steps to solve all addition and subtraction problems.

The fourth principle reminds teachers of students with learning disabilities to instruct students with clear and explicit language. Rosenshine and Meister (1992) point out that efforts to teach higher level thinking often fail; due, in part, to inadequate instruction. During instruction, the teacher's questioning language needs to be explicit and clear, following a pattern that guides thinking. Carnine (1997) reminds us that teacher directions and questions are usually the most important guides for student understanding.

To help teachers explain the schema diagrams effectively, Jitendra and Hoff (1996) developed a teaching script to go along with each schema diagram and checklist. The strategy itself has repetitive elements to help students learn. In the schema-based approach to math problem solving, the students proceed through the same sequence of steps for each word problem by following the steps on the FOPS checklist (see Appendix C). First, students read and retell the problem. Then they Find the problem type and Organize the information by highlighting facts in the sentences and mapping the numbers and labels into the diagram. Next, they Plan what to do, choosing the operation. Last, they Solve by calculating the answer and checking to see if it makes sense.

Carnine's (1997) fifth principle is to provide practice and review to facilitate retention. Teachers should provide intensive strategy instruction initially, and then continue with spaced practices. He says this mix leads to automaticity and retention. Corrective feedback during initial instruction and during practice is extremely important and supported by a large body of literature (Carnine, 1997). When students make errors, the preferred strategy is to repeat the original strategy instruction. Only when errors persist, even though appropriate instruction is being delivered, should an alternative strategy be tried (Carnine, 1997).

The researchers agree that learning to use a strategy to solve mathematical word problems is a valuable goal for all students. Initial strategy instruction relies on teacher directed lessons as the students learn the new skill. Later lessons must support students in leading the problem solving process, as they learn to follow the patterns in the checklists and diagrams.

Discourse is a term that encompasses all the talk that occurs in a classroom. There are two main patterns of instructional language in classrooms, and both kinds are valuable. Traditional patterns of instructional talk have a three-part sequence: Initiation, Response, and Evaluation (Cazden, 1997). This pattern is appropriate when teaching a new routine, strategy, or process and it allows the teacher to control and guide the discussion toward the intended learning goal. In the introductory lessons about the strategy, this approach is most appropriate.

The non-traditional pattern seeks to contrast with the teacher controlled traditional pattern because the goal of instruction is a new goal – to encourage inquiry by students that leads to discovery of knowledge. Students need to learn this pattern of discourse to participate in a problem solving conversation.

Non-traditional classroom discourse is significantly different in several ways from traditional discourse patterns:

1. Teachers and students accept alternative student answers, and ask for comparisons with supporting reasons. Explanations are as highly valued as answers. Listening and responding to peers is expected.
2. The teacher seeks to understand each student's understanding. This requires that teachers have an in-depth understanding of the subject area.
3. The teacher faces the dilemma of honoring child logic and teaching conventional knowledge. In mathematics and the sciences there is a strong argument for conventional knowledge as a basis for more complex understandings. (Cazden, 1997)

The teacher must teach the students these new values and the new skills in order to support non-traditional discourse in the classroom.

Cazden (1997) offers examples of well-planned designs for implementing non-traditional discourse. Some such programs are: Clay's *Reading Recovery*, Palincsar's *Reciprocal Teaching*, and Brown's *Community of Learners*. All of these designs embrace Vygotsky's (1978) "zone of proximal development" as a guiding principle and incorporate the concept of scaffolds to support students as they learn. Vygotsky conceptualized development as "a complex dialectical process, characterized by periodicity, unevenness in the development of different functions, metamorphosis or qualitative transformation of one form into another, intertwining of external and internal factors, and adaptive processes" (p. 73). Children are part of the creation of their own learning. The interactions of the environment, the child, and the interpreters of experience result in changes in the child and in others in the environment. This dialectical model of learning recognizes that children will learn at different paces. Discourse patterns that include scaffolds in their design support students as they learn in their own way.

Students need to learn to engage in discourse. "Instructional Frames" is Andersons' term for a set of sequenced, predictable activity structures (Cazden, 1997, p. 108) that help teachers and students to proceed automatically through a lesson, free to concentrate on the instructional discourse. Cobb and colleagues (as cited in Cazden, 1997) described a second grade math teacher's explicit expectations for working with a peer partner in the classroom. Students were expected to explain their math thinking and their partners' math thinking, listen to and try to understand their partners' explanations, challenge explanations that did not seem reasonable, and agree with their partners on an answer and solution method. These social norms were not taught as a list of rules, but were taught in an ongoing manner as specific incidents provided occasions for a discussion of the expectations.

Cazden (1997) includes Cobb's description of a discourse pattern between a pair of students that he termed "multi-vocal". In this pattern, both children simultaneously attempted to advance their own perspective. Both students explained and challenged in turn. Both ideas were discussed at the same time. A more traditional discourse would be more linear – one method would be discussed, then the next would be examined. Students engaged in discourse will not necessarily engage in formal, linear exchanges. Students devise their own patterns of discussion.

Collaborative problem solving requires students to know how to talk about mathematical problems. They must also show respect for each other and each other's ideas. They need to listen, try to understand, and to politely challenge explanations that seem unreasonable. Finally, they need to agree on an answer or a solution method and be able to explain and justify their work. The non-traditional discourse pattern offers both social and academic support to students.

Establishing conversations about mathematical problem solving requires that the teacher model the interactions for the student to emulate. Cohen (as cited in Cazden, 1997) describes three teacher actions to establish these "collaborative norms". First, teach students the skill of "explaining by telling how" (p. 151). The skill is linked to these key phrases: It is your right to ask for help. It is your duty to give help when asked. You explain by telling how. Second, the curriculum and tasks must openly require multiple skills. The third important teacher action is to "assign competence" to children by giving "specific, positive, public evaluation and recognition" (Cazden, 1997, p. 111) when they contribute one of the multiple skills to their group. Working to establish equity in classroom discourse in the elementary grades will hopefully establish patterns of respect that last throughout each child's life.

When teachers decide to conduct discourse with their students, they integrate many influences. Some influences are the conscious ideas that teachers plan and bring into their classrooms. Other influences are generated in the moment, in the midst of an interaction during class (Cazden, 1997, p. 145). These decisions are a function of who we are and what we know. The more we understand about different learners, about effective instructional strategies, about the elements and effects of discourse, about the topics we are teaching, and the way people learn, the better our decisions will be.

As Cazden (1997) acknowledges, increasing challenges created by the new conception of curriculum are reflected in national and state standards. The emphasis has moved away from "products, facts, or procedures to be learned" to "processes and strategies for learning and doing" (Cazden, 1997, p. 5). As the students learn to engage in discourse to solve word problems, they also learn to follow steps in a strategy and to communicate in an organized way.

The National Council of Teachers of Mathematics published the *Professional Standards for Teaching Mathematics* in 1991. This book describes teacher actions that "advance the vision of high-quality mathematics instruction for every child" (NCTM, 1991, p. viii). Part of high-quality math instruction is classroom discourse about mathematics. The teacher's job is to direct the conversation, so students will learn mathematics and learn to communicate in a mathematical way (NCTM, 1991). Students should experience a math classroom in which "reasoning and arguing about mathematics is the norm" (NCTM, 1991, p. 35) and where all ideas are treated respectfully by the teacher and the peers. Math teachers must work to develop skillful questioning patterns to guide the students' thinking and the direction of the discourse. *Why* questions stimulate mathematical reasoning, and can be asked in response to both correct and incorrect proposals (NCTM, 1991). The teacher decides when and what to ask by listening carefully to students' ideas and focusing the discourse by picking up on some points and leaving others alone (NCTM, 1991).

It is important that all students in the class participate in the discussions. The teacher must find ways for all students to reason mathematically and explain their thinking, including "means that are pictorial, concrete, representational, or oral" (NCTM, 1991, p. 36). The students should actively engage in "making conjectures, proposing approaches and solutions to problems, and arguing about the validity of various claims" (NCTM, 1991, p. 45). It takes time and practice for students to adopt these behaviors.

Students learn how to engage in math discourse as a result of teacher actions that establish the classroom environment, create appropriate mathematical tasks, and ask stimulating and guiding questions. To support student discourse the teacher must listen carefully to students' ideas, insert questions to guide the discourse, and press students to seek connections. By providing models of good arguments and poor arguments, the teacher can clarify what a math argument should prove. Teachers can help students by reminding them of math concepts that apply to the problem. Teachers should gradually add conventional math language to enhance the meaning of the discourse, so that by high school formal math terminology is appreciated for its precision (NCTM, 1991).

When teachers communicate enthusiasm for mathematics and involve students actively in doing mathematics, they help to develop a mathematical disposition in their students (NCTM, 1991). Students who are disposed to doing math "demonstrate confidence, flexibility, perseverance, curiosity, and inventiveness in doing mathematics through the use of appropriate tasks," and they engage in mathematical discourse (NCTM, 1991, p. 104).

The National Council of Teachers of Mathematics (1991) urges teachers to shift toward this vision of teaching math so that all American K –12 students can develop mathematical power. (NCTM, 1991).

RESEARCH DESIGN AND METHODOLOGY

Participants and Setting

I teach in a public school located in semi-urban suburbs around a small northeastern city. The density is urban, many homes are doubles, and the yards are small. The community consists mainly of middle or working class families. Increasing numbers of minority students and students who are English Language Learners are making the student body more diverse. Our school qualifies for Title I funding for reading assistance.

I am one of three special education teachers in my building, which is one of three elementary schools in the district. In our building we have about 40 identified special education students in grades K - 5. One teacher is responsible for all identified special education students in grades K - 2. I am one of the other two teachers who share responsibility for the identified students in grades 3 - 5. The students on our class lists can have a range of disabilities including mental retardation, learning disabilities, other health impairments, emotional disturbance, language learning disability, and autism spectrum disorders. One challenge we face is the variability of needs and the range in severity of the disabilities. If we think a student is not making progress in our setting, we can request a reevaluation and then convene the Individual Educational Program (IEP) team to consider a change in placement. One of the students involved in my study was referred for reevaluation because of his inconsistent performance and unusual behavior across all his educational settings. Because the process takes from two to three months, the reevaluation was not completed during the course of the study.

I had fifteen special education students on my class list, but each day I taught twenty-one identified students in my classroom. This sharing of students in third, fourth, and fifth grades allowed us to instruct students at their level in reading and math. By pooling our staff resources, aides could provide daily in-class support for regular instruction for some students, while the two special education teachers provided pull-out math or language arts instruction for those students who needed it. The time I spend teaching in the special education setting and supporting in the inclusive regular education setting changes each year, depending on the students' needs. This year, I spent only ½ hour per day supporting IEP students in the regular education setting.

My study focused on four students who received math instruction in the special education setting. I had taught math to these students for part of their third grade year, and I taught them during their fourth grade year as I conducted my research. All of the students in my class had Individual Educational Programs (IEPs) because they had been identified through an extensive testing process as needing Special Education in order to learn. In order to respect each student's privacy, each student is referred to by a pseudonym. I described the type of learning problems that were represented in the group through general description, and displayed each student's academic skill levels on Table 1 and Table 2.

Table 1: *Student Profiles: Academic Skill Levels – September, 2004*

Name	*Math*	*Reading*	*Spelling*	*Writing*
Terry	Gr. 2 - 14 dpm Gr. 3 – 8dpm Gr. 2 – 100%	Gr. 3 - 99% - 86 wpm Gr. 3 Comprehension 83%	Gr. 3 - 98%	Gr. 3 - 42 words in 3 minutes
Pat	Gr. 1 - 10 dpm Gr. 2 – 8 dpm Gr. 2 – 84%	Gr. 2- 99% - 93 wpm Gr. 2- Comprehension – 80%	Gr. 3 - 94%	Gr. 3 – 38 words in 3 minutes
Ben	Gr. 2 - 5 dpm Gr. 3 - 3 dpm Gr. 2 – 92%	Gr. 2 - 96% - 100 wpm Gr. 2- Comprehension - 95% Gr. 3 – 71 wpm	Gr. 4 - 96% Regular ed.	Gr. 3 - 16 words in 3 minutes
Andy	Gr. 2 - 8 dpm Gr. 2 - 92%	Gr. 1 - 97% - 17 wpm Gr. 2 —Comprehension - 80%	Gr. 3 - 83%	Gr. 3 - 8 words in 3 minutes

Table 2: *Student Profiles – Academic Skill Levels – January, 2005*

Name	*Math*	*Reading*	*Spelling*	*Writing*
Terry	Gr. 2 -25 dpm Gr. 3- 17dpm Gr. 3 –90%	Gr. 3 - 99% - 100 wpm Gr. 3-Comprehension- 85%	Gr. 4 - 97% Regular ed.	Gr. 3 - 47 words in 3 minutes
Pat	Gr. 1 -18dpm Gr. 2 -16dpm Gr. 2 – 94%	Gr. 3 - 94% - 66 wpm Gr. 3-Comprehension- 69%	Gr. 3 - 82%	Gr. 3 - 42 words in 3 minutes
Ben	Gr. 2 -15 dpm Gr. 3- 12 dpm Gr. 2– 95%	Gr. 3 -96% - 80 wpm Gr. 3 - Comprehension 84%	Gr. 4 - 94% Regular ed.	Gr. 3 - 24 words in 3 minutes
Andy	Gr. 2 -13 dpm Gr. 3- 12 dpm Gr. 2- 98%	Gr. 1 - 97% - 37 wpm Gr. 2 -Comprehension- 80%	Gr. 3 - 90%	Gr. 3 - 13 words in 3 minutes

I included data on reading ability because reading and understanding questions is an important part of solving word problems. All the students in this group could decode at the second grade level, according to weekly curriculum based assessment probes. All the students had difficulty with reading comprehension. The comprehension scores indicate the percentage of questions the student could answer correctly at their instructional reading level. At the first and second grade levels, most of the questions were factual, sequential, or cause and effect questions. At the third grade level, the comprehension questions included more inferential reasoning.

At the beginning of the school year, most of the students could do basic math arithmetic calculation at the second grade level, though their fluency was poor. This included addition and subtraction facts, simple multiplication facts, and adding and subtracting with renaming one time. Scores below 10 digits correct per minute (dpm) on timed grade level math probes indicate the frustration level. Scores of 10 –14 digits correct per minute indicate the instructional level. Scores of 15-20 dpm indicate the independent level. In September, three students scored at the frustration level on the 2nd grade probes, and the one student scored at the instructional level for 2nd grade. Throughout the study, students practiced the computation skills and were tested on timed probes two or three times per month. I used this measure to indicate progress in mathematics, based on progress monitoring benchmarks recommended by the Pennsylvania Training and Technical Assistance Network (PATTAN) (see Appendix E).

Near the end of the research study, a new student entered the class. Although her computation skills were similar to the other students in the class, she had missed the intensive strategy instruction and all the practice in working with a partner to engage in dialogue to solve math problems. In order to include her in the class and still proceed with my study, I had to make some adjustments. To do this, I taught a series of lessons focused on money. I incorporated the Change and Group diagrams in this context. I did not seek parental approval for her participation in the study.

During the last week of my data collection, another new student joined the math class. She had good computation skills and was able to solve word problems at the second grade level, but she had also missed the strategy instruction and the practice dialogue with a partner to solve word problems. During partner work, I grouped the two new students with each other and spent time teaching them to work together. Unfortunately, this prevented me from observing the other students as they worked together. I was not comfortable segregating the two new students during partner work, so I began to vary the groups by randomly choosing names for partners.

The addition of the new students seemed to interrupt the flow of everyone's progress as the new social relationships developed. This time of day, right after lunch, brought these six students together for the first time during their day. They came from three different homeroom teachers and were in three different reading groups. The two girls were new to our class and new to the school district. It took time to integrate them into the math community that we had built. Some pairs did not work as well as others. Then in January, another new student joined the class. He already knew all the other students, so his integration was not as difficult. But he made the seventh member of the class. Now groups were not all equal. Group work was not the joy for the students that it had been earlier in the study. I found that I needed to experiment with the group members to find successful combinations of students. Eventually the groups began to function again. Data about these three new students is not included in my study beyond this acknowledgement of the possible effects it had during the final weeks of the research.

Many factors influence how students learn, not only academic skills. The social pressures were real and significant. Donahue (2002) reports that "social responses that seem inappropriate to adult observers may actually serve strategic social purposes for students" (p. 253). She found that multiple studies report that students have accurate personal social knowledge of how their peers will react to them. This influences their choices during discourse.

It took time for the students in my math class to determine what kind of responses to risk in this small group setting. Two students had Language Learning

Disabilities and received Speech and Language Services. All of the students had learning disabilities as part of their diagnosis. Three of the students had difficulty with attending or maintaining focused attention. One of the students took medication to improve attention. One student was being reevaluated for emotional problems that interfered with his achievement. In all subjects his performance was inconsistent and highly variable. This pattern held true for his math performance.

Some students worked best when written work was the focus, and others were very slow in completing written assignments. Some students were intuitive and grasped concepts, but had difficulty with details. Other students were methodical and understood patterns and steps to follow, but struggled to grasp the big picture. This small group of students was quite diverse in their strengths as well as in their weaknesses. But, they proved to be willing co-researchers. They had a positive reaction to being part of the study. They learned to take risks and get along with each other as math partners. When I asked them what helped them understand, they gave me honest answers.

Baseline Data

During the first week of school, I tested the students to gather baseline data in order to monitor their progress for their Individual Educational Plan (IEPs). Throughout the year I tested for data regularly for each of these areas: reading fluency and retelling, writing fluency, and fluency on math computation probes. Although the math probes do not directly measure problem solving, the fluency measure, Monitoring Basic Skills Program- Basic Math is a standardized form of curriculum- based assessment developed originally by Deno and Fuchs (Fuchs, Hamlet, & Fuchs, 1990). Over fifteen years, Deno teamed with other researchers to improve the measure. It shows student progress in math by sampling fluency on mixed computation problems at their instructional level. The students' scores on progress monitoring probes are reported on quarterly report cards. Standardized benchmark levels were used to judge student academic progress using the Basic Math CBA probes (see Appendix E).

Special education researchers define effective instructional strategies as those approaches that result in increased academic performance (Vaughn & Linan-Thompson, 2003). To measure academic growth, progress monitoring tools such as curriculum-based assessment (CBA) probes are used in each area. A probe is a short, timed test used to measure accuracy and fluency. A typical math probe is a page of twenty-five varied math problems representative of skills that should be mastered by the end of that grade level. Students are given two to five minutes to work on the problems. Each correct digit in the answer is recorded as a point. With long multiplication, division, and fraction problems, digits in intermediate steps are awarded points. The scores are recorded and charted to indicate if the students are making progress toward their math goals. When data on student progress are monitored frequently, adjustments can be made in each student's program to foster success and progress. This progress monitoring approach makes interventions responsive to each student's needs by quickly identifying ineffective instruction. (Deno, 1985; Good & Kaminski, 1996, as cited in Vaughn & Linan-Thompson, 2003). Changes in the instructional program are then implemented to maximize student learning. Progress monitoring guides the teacher to select and implement effective teaching strategies.

I began to formally collect data after Moravian College's Human Subjects Internal Review Board approved my research proposal (see Appendix F). The board reviewed the plan to ensure safety and confidentiality for the children involved in the study. While I waited for approval, I taught word problems as part of the math curriculum. These word problems asked questions that used real life tools such as clocks, calendars, rulers, and money. Students began to work in pairs to solve problems using these tools in math centers. I recorded my observations of these early interactions in my field log. These records served as baseline data to compare with later partner interactions and are reported in my story. During Math Center time, I guided student pairs as they tried to solve the math puzzles. We checked the results at the end of each session. No formal instruction was involved. I wanted to see if they would apply their knowledge to novel problems as they worked with a partner. I received my principal's permission to conduct the research project once the HSIRB approval was secured (see Appendix G).

Intervention

I introduced the first schema strategy early in October, after all the parent permission forms were returned (see Appendix H). First, I presented the three schema diagrams – Change, Group, and Compare - in a general way. I explained that trying these diagrams was part of my research, and I hoped to find out that the strategy helped them talk about and solve word problems. I told the students that all addition and subtraction problems could be represented using these three diagrams. We would learn how to put numbers from word problems into the diagrams (mapping), and then write a math sentence from the diagram. Each schema diagram has a problem solving checklist to guide the students in its use (see Appendixes B, C, and D). The problem solving checklists are based on the four steps for solving word problems indicated by the acronym FOPS. (see Appendix I).

Teaching the Schema Representational Strategy was central to my research. The FOPS problem solving steps, and the metacognitive guiding questions that were part of it, would become the common language for the students' discourse about problem solving. I introduced the strategy in gradual phases. First, I taught the Change schema diagram and checklist. After practice with the Change diagram, I introduced the Group diagram, and then two-step problem solving. Although I did not have time to introduce the Compare diagram during the course of my study, I did teach the Compare strategy to the students as I was writing the findings from my study.

Each strategy lesson included talking-through and mapping some problems together, then three or four problems for the students to solve with a partner. I observed, coached, and collected their work samples. Quizzes were initially independent work, but later the students preferred to work together. I allowed it since I wanted to foster discourse about math problem solving. I corrected the students' work with them and I recorded their percentage correct on quizzes. To determine how accurately students solved problems on their own, I also assigned independent problem solving quizzes.

During the strategy lessons, I gave students opportunities to take the teacher role in guiding us through the problem solving steps and the mapping. In their partner groups, they began to use the diagrams and checklists to explain and discuss their decisions at each step. I coached, modeled, guided, and praised the use of specific math language in their conversations.

One day per week, the students worked with word problems in a math center. The problems were math puzzlers that we called thinker problems. They did not usually fit the problem types represented by the schema diagrams. Students could use the math tools in the center to figure out the answers with their partners. I observed them, coached them, and recorded my observations in my field log.

Student Surveys and Interviews

Before beginning the research study, the students completed a survey about math problem solving (see Appendix J). The questions were designed to measure their attitudes towards math problem solving and to determine how they perceived themselves as problem solvers. I read it aloud, and they circled responses on a rating scale. The students chose from four categories to describe themselves as problem solvers. As Arhar, Holly, and Kasten (2001) suggest, I kept the questions short and limited the scale to "no less than three and no more than seven" choices (p. 151). One question asked them to write reasons to support their opinion of the importance of problem solving. In the middle of the study, I conducted a group interview with the students to find out their response to using the schema diagram strategy, and to find out how they felt about talking with a partner to solve word problems. I reported on this informal interview in my story. Near the end of the research study, I interviewed the students individually and recorded their responses on the interview form (see Appendix K). The interview form was based on the Mathematical Problem-Solving Assessment-Short Form (MPSA-SF) developed by Montague (1997). Each student solved word problems as part of the interview. I encouraged them to explain aloud as they worked and I took notes on their strategy use and accuracy. If I coached them, I recorded the conversation we had. They repeated the original survey questions as part of the interview at the end of the study.

Field Log

I recorded my participant observations of the problem solving strategy classes and the Math Center activities in a field log. Since I was always actively involved during these activities, I am considered a participant observer (Arhar, Holly, & Kasten, 2001). During math class, I jotted down pieces of conversation when I could. I took notes on the lesson materials as I taught. When I reviewed the students' work at night, I added notes about what I recalled from the lesson and reflections about their work – errors and successes. Ely, Vinz, Downing, and Anzul (1997) define field notes as "the written record of the data as shaped through the researcher's eyes" (p. 15). Sometimes I relied on my memory of the class, my lesson plan materials, and student work to reconstruct the class at night after I got home. I did not record descriptions of classes when I taught the Math curriculum. My participant observations focused on the specific strategy instruction or on the students' interactions and discourse as the partners worked together to solve word problems or thinker problems. I described in detail teacher and student actions and reactions during strategy instruction and during word problem solving activities. I recorded the coaching they provided to each other or that I provided to them, including their written responses and conversation. I put brackets around observer comments that are my personal reflections about the students' progress and behavior, and about my progress in teaching the Schema strategy.

Student Work Samples

I saved many examples of student work in my log. I wrote notes on the work papers, indicating the coaching the students received, their response, and their level of understanding. Students took quizzes each week. I kept an individual chart of each student's quiz scores and the types of problems on each quiz. To get more information from the quizzes, I evaluated by giving three points for each problem: one for representing the problem correctly - by mapping correctly or by setting up the correct number sentence, one for choosing the correct operation, and one for getting the correct answer. I hoped to see improvement in these areas, especially in the representation of the problem.

The students wrote the answers for the Math Center puzzles in their math journals. I tabulated the percentage correct for each student. I wrote detailed observations of student pairs as they worked to solve five word problem puzzles together. These recorded interactions and my observer comments were included in my field log.

Trustworthiness

In order to verify my interpretations of the data, I needed to compare my judgments with other sources. I could do this by triangulating multiple sources of data (Arhar, Holly, & Kasten, 2001). I reflected in writing when things were going right and when things were going wrong. I responded to student confusions by asking them for feedback on the day's lesson, and recorded that information in my log. When students made requests, I considered their ideas and sometimes changed my plan. I was willing to adjust the schedule and grouping in response to student input, although not every student request was honored. The impact on all members of the class was considered. I conducted an informal group interview with the students to check conclusions. I thought some students might feel intimidated in a one-to-one setting with a teacher, so I also conducted a group interview. The formal interview at the end of the study gave students a chance to express their opinion in private.

Surveys provided another way to check my ideas. Students completed an anonymous survey at the beginning of the study in September and again in December.

Written student quiz and math probe data provided information about what students knew and could do. I compared their oral work with their written work. I read and reread my log to examine my conclusions and to find data to support my assertions.

I used the data to create various narrative forms describing my students and their experiences in math problem solving. I used participant checking to see if students recognized truth in my portrayals. I let them read what I had written and they told me that they felt the characterization was fair.

Throughout my study, I shared my notes and my interpretations with my teacher researcher group. Both teachers in my researcher group are colleagues in my school building, and both are regular education teachers. They helped me understand what regular education students typically could do. Sometimes this helped me see my students as less competent than I thought, and other times, it reassured me that the students were gaining valuable skills.

MATH STORY

I introduced the idea of the research study to my students as I prepared to pass out the permission form. I explained that I am a student as well as a teacher, and, that I wanted to learn how to be a better math teacher. I had read about some ideas for teaching math, especially about solving word problems. I said I hoped they would be interested in helping me try out this new idea. Their job would be to tell me about what helped them understand and what did not help. I would be writing a report about what we did, telling about things that worked and things that didn't work. During math classes, I would be listening and watching them work, and sometimes I would be writing down what they said and did. I would also write down things I said and did. I told them that I would not use their real names in the paper. I would make up fake names for them. They got excited about that comment – they wanted to know if they could pick their own names. I said that I had already picked the fake names. Then they wanted to know what their fake names were, but I said they were secret from everyone. They accepted that. I told them about the interviews. I would be asking them questions to find out how they think about math problems. I said I wanted them to show me and tell me how they figure problems out as part of the project. They wanted to know if they had to write a report, too. I said no, I had to do that part. They would all do the same work in math even if they did not want to be part of the study. Being in the study just meant that their scores and ideas would be included in the report.

I read the permission slip to them, but I used shorter sentences and simpler words for the formal language of the letter. I asked them to get their parent's signature and return the paper to me by the end of the week. I passed out the letters in big envelopes that would not be easily lost. All four students got their parents' permission to be part of the study. We were ready to get started!

Math Meeting

I taught the third grade Saxon Math curriculum to this group of fourth grade students. Each day we had a "math meeting" time. During the daily meetings, the students practiced a variety of math skills related to real-life math, such as telling time and counting money. Through the math meeting activities, I modeled how to use the tools that were stored in the math center; including rulers, calculators, play money, and hundreds charts.

They started solving math puzzles in the Math Center activities before I taught the schema strategy steps for solving math word problems. They had previous experience with problem solving from third grade, and they had practice from four math meetings this year. I used these early math centers as a baseline for the growth I hoped to see in their discourse and problem solving skills.

Math Center

I introduced the first Math Center activity by showing them where all the math tools were stored in the math center area. I had the materials stored in matching boxes on a wire shelving-unit in the back of the room. I gave each student a math journal for their work and their answers. I assigned partners and explained that they would stay with the same partner for a while. Later, they would decide if they wanted to take turns working with different partners. The partners would work together to figure out the answers to math puzzles.

I told them that real mathematicians often work together to solve problems. Math professors get together to discuss problems and brainstorm different approaches to solving the puzzles that the problems represent. They try out their ideas and learn from each other. They talk and ask each other questions. Sometime they disagree. All of that conversation helps them get new ideas or fix up the idea they had. That is what we would be doing this year. We would practice working and thinking like mathematicians.

I introduced the math center problems as puzzles. I got the puzzles from a problem solving kit, Techniques of Problem Solving - Deck A, published by Dale Seymour Publications (1980). Kit A contains problems based on third grade level skills and concepts. Each problem is presented on a four by six card with a picture or graphic clue.

To solve these puzzles, I told the kids, you might use tools from the math center. I explained that these math center problems could be solved in more than one way; so two different approaches might both get the right answer.

I handed each pair some word-problem cards to work on. They asked me which problem they should do. I told them in any order, just be sure to put the answers in their journals next to the card number. Dewey (1938) used strong language when he wrote:

> There is no point in the philosophy of progressive education which is sounder than its emphasis upon the importance of the participation of the learner in the formation of the purposes which direct his activities in the learning process. (p. 67)

The freedom to choose the sequence in which they solved the math puzzles allowed the students to direct their activities in during math center time.

I guided the students in using the center tools to solve their puzzles. Sometimes I had to suggest a tool that fit the task. The word "between" in the directions had a specific meaning that confused the students. They needed guidance to interpret between questions so they could get the right answer. Other terms were not confusing, such as "odd and even numbers". Questions that asked for comparisons were not solved correctly, and the two-step problem was only partially solved. As an example of the guidance they needed, I have included some of our dialogue from the first set of center puzzles:

I checked over the work Ben and Andy had done. I found they had two errors, so I suggested they rethink those. They reread the first card and Ben said, "We added them to get twelve. Seven and five is twelve."

I responded, "Yes, 7+5=12, but do you need to add?"

Ben continued, "We counted 7 and 5 is 12."

I said, "Wait. Let's look at the question." The card showed two groups of things, and the question asked for how many more one group had.

I got chips out to represent the things in the groups and showed them 7 in one group and 5 in the other. Then I lined up the two rows of chips to compare them and then I asked, "How many more does one row have than the other?"

Andy answered, "Two more."

I said, "Yes. That is what they wanted to know." You need to check what the question is asking."

Ben said. "Oh, two more!"

The students struggled to represent this problem in a meaningful way. They did not pick out the key words "more than" that indicated subtraction, and they did not visualize the compare situation. Even when I pointed out that the problem compared two groups, they did not connect compare with subtraction.

While I was teaching math, I also had other students coming and going for language arts instruction. The math class met from 12:40 until 1:45. Third grade students came to my class for spelling and English from 1:20 until 2:30. By this time, my assistant was gone for the day, so I needed to let the math group work on their own, while I started my third grade students on a spelling activity. When I got back to the math group, Ben and Andy said, "We're done."

"This is not a race," I said. "When I pass out the problem cards, I just randomly hand them out. I do not check that each group has three hard ones and three easy ones. I just pass them out. You can't tell if one group has all hard ones and one group has all easy ones. You have to slow down and think about each one. Don't race to get done first. That's how you make mistakes on things you know."

As a baseline, I could see a weakness in understanding the questions. Both groups had made errors because they did something with the numbers in the problems that did not answer the question asked on the card. Another weakness was in precision. On the calendar, alphabet chart, and hundred chart questions, their work correctly addressed the question that was asked, but they struggled to get the precise answer. Knowledge of vocabulary was weak. They did not clearly understand how to apply the concepts of *between* and *compare* in a math situation. The students could not explain the steps or clues that led them to their solution, even if their answers were correct. I saw students choose to add, but they could not tell me any reasons why they chose to add.

I examined my own explanations. I noticed that when I coached a group through a puzzle, my explanation was very specific to the situation. Each situation was presented as a unique one. These puzzle problems would be challenging because they would present unique situations and require independent mathematical reasoning and application of their math knowledge.

The students got along well and managed the passing out and cleaning up of journals, question cards, and center materials independently. They read the problems together and picked tools to use. They were excited and eager, judging by comments like, "Is math over already?" and "Can we do Centers today?"

During the first week of the study, the students' behaviors, scores, and comments in class and during math conversation gave me some insight into how they felt about themselves as problem solvers. I wrote these short poems to illustrate their feelings and their cognitive styles as they attempted to solve mathematical word problems. I used quotes and notes from early entries in my field log.

Figure 1. *Character Poems*

Terry

Focus.

Concentrate.

I want to get it right

Quiet.

I'm not sure.

Is this right?

Do it this way.

Remember the pattern?

Why didn't it work?

This makes me mad.

Ben

I thought we were doing something else.

Why are we doing this?

I thought we were doing fourth grade work.

I don't want to.

Can we play with Wikki Stix?

Can we play a game?

Can we get prizes for the game?

This is confusing.

I don't feel good.

<u>Pat</u>

What did you say?

What adds up to 10?

I don't know. *Shrug.*

Are we still partners?

Can I get the counters?

I did that!

I got it!

<u>Andy</u>

Watching and thinking.

Not really fast.

Trying to be faster.

Show me.

Let me do it.

Now I get it.

It makes sense to me.

After I wrote these poems, I wanted to see if the students could recognize themselves or each other in my descriptions. I spoke to each student. I explained that while I was reflecting on our problem solving journey this year to write my paper, I wrote a poem to describe each of them at the beginning of the year, when we first did Math Centers. I gave them a copy of the poems and let them read on their own. Every student correctly identified himself or herself and each other. I asked the new students to identify themselves in poems, too.

Introducing the Change Strategy

The next week, I introduced the schema strategy to the students. I displayed the three diagrams we would learn. I had big posters of each diagram on different color paper. I showed them the Change, Group, and Compare diagrams. I explained that these three diagrams could be used to solve addition or subtraction problems. I briefly stated the different types of problems as they applied to each diagram. The Compare diagram is used when you compare the amount you have in two different groups, to find out how many more or how many less. The Group diagram is for adding small groups together to see how many they would make as a big group. The Change diagram is for problems that have a beginning amount, then a change happens, and there is an ending amount.

We would learn to write, or map, numbers and labels in the diagrams to help us solve word problems.

Last, I displayed another poster. This one was white and it listed the steps for solving word problems. This was the FOPS checklist. Each letter stands for a step in the word solving process.

F = Find the problem type.

O = Organize into the diagram.

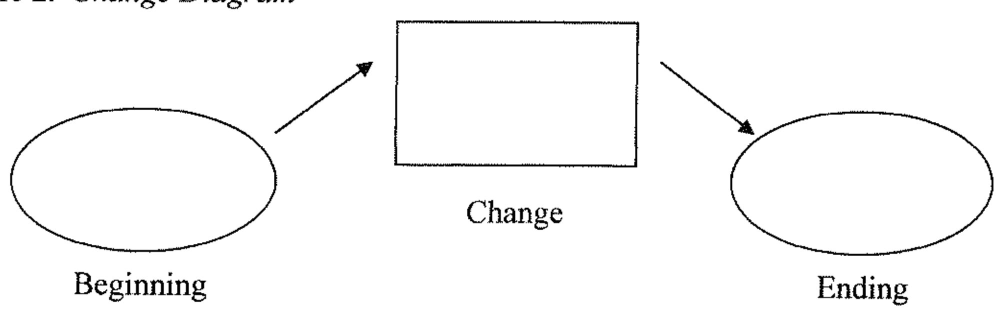
P = Plan to add or subtract.

S = Solve and state the answer.

We would use these same steps for every diagram so we would have lots of practice with them.

We started with Change stories, so I kept just the Change diagram on the board. Then I gave each student a FOPS checklist and a Change diagram, both in plastic sleeves. I put a can of whiteboard markers in the center of the table for their use.

Figure 2. *Change Diagram*



To illustrate how I instructed the students in the strategy, I include a reconstructed account from my log and from the scripted teacher materials that I used to pattern my instruction.

I explained: “Look at your Change diagram. The Change diagram has a beginning, a change, and an ending. We will read some math stories and figure out where to put the numbers to show how many they had at the beginning, what changed, and how many they had at the end.”

“One other thing!” I said excitedly to get their attention, “In a Change problem, each sentence describes the same thing. The focus of each sentence in the story is only one item, like video games, video games, and video games.” Ben repeated that phrase: video games, video games, video games and told us how he loves to play video games!

I passed out a paper with three Change stories on it and told them to pick a marker from the can. I told them to look at the Change story checklist. The story checklists have only two steps, because there is no need to plan and solve. The answer is part of the story, and the task is only to map the parts of the problem correctly.

Figure 3. *Change Story Checklist*

Step 1. Find the problem type.

- ☐ Did I read and retell the story?
- ☐ Did I ask if it is a change problem? (Did I look for the beginning, change, and ending? Do they all describe the same thing?)

Step 2. Organize the information using the change diagram.

- ☐ Did I underline the label that describes the beginning, change, and ending and write in label in the diagram?
- ☐ Did I underline important information, circle numbers, and write in numbers in the diagram?

Step 1: Find has two parts. The first part is to read, then retell the story. I modeled by reading:

Jane had 4 video games. Then her mother gave her 3 more video games. Jane now has 7 video games.

I retold by restating the problem without looking at it, but stressing the important information. The second part asks if it is a Change story. The metacognitive guiding questions ask, "Did I look for the beginning, change, and ending? Do they all describe the same thing?"

Although it is in Step 2: Organize, that you are supposed to highlight important information and write the labels and numbers in the diagram, I ended up doing both Step 1 and Step 2 at the same time. They could not understand the question – Does it have a beginning, change, and ending? I had to guide the students through each sentence of the story in order to model how to identify the key words and semantic elements that indicated the beginning, change, and ending. I had to teach them to look at each sentence for these three parts. That led me to highlight the information in each sentence as I explained; and as I explained, we mapped the information into the diagram to emphasize where it belonged.

First, we looked to see if the question was asking about only one thing. They eagerly identified that the video games were the focus in all the sentences of the story. We highlighted video games in each sentence. We wrote video games in all three spaces.

Next we had to decide if there was a beginning, a change, and an ending by considering each sentence by itself. The first sentence said: Jane had 4 video games. I circled had 4 video games and modeled by thinking aloud –had tells me that in the beginning this is what she had. Had is the key word that tells it was in the past, at the beginning of the story. We all wrote 4 in the beginning oval of our diagram. It now read: 4 video games.

Then we had to decide if a change happened in the story. I told them to look for an action word. We considered sentence two. It said: Then her mom gave her 3 more video games for her birthday. I modeled by circling the important words (gave her three more). I asked them, "What happened in this part of the story?"

They all spoke out to answer that mom gave her 3 more games.

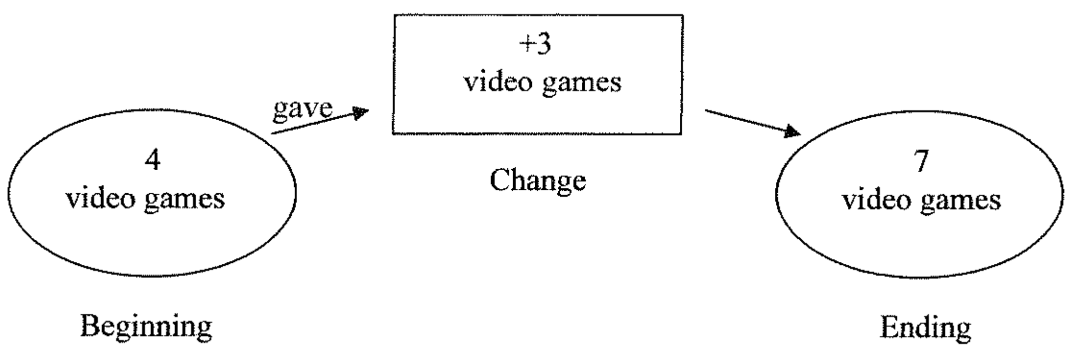
I underlined gave. Then I said, "This is the change that happened. Now we know it is a Change story. They underlined gave on their story sheet with their markers. So we wrote in 3 video games in the change box.

Ben spoke up, "Add."

I paused. "OK, Ben. If her mom gave her three more, we would add three because it gives her more video games. You can write a + sign above the change word, gave, in the story; and we will write a + sign in front of the 3." Now it read + 3 video games in the middle (change) box.

Then we looked at the last sentence in the story: Jane now has 7 video games. We needed to find the key words. I modeled by underlining "now has 7". I explained, "Now has tells us this is the end of the story. So we have a beginning, change, and ending. We all wrote 7 in the ending oval in the diagram. We had underlined and circled the key words and information and mapped the facts into the Change diagram.

Figure 4. *Completed Change Diagram*



I said, "Lets see if our story makes sense. I can read the diagram as a math sentence." I pointed as I read: "4 video games + 3 video games = 7 video games. What do you think?"

The students said, "Yeah. That's right." Some just nodded.

"Ok," I said, "Now let's check if we did the math properly. Does 4+3=7?"

They all agreed it was correct. The number sentence told the same story.

The students were excited about writing with the markers on the plastic covered diagram and checklist. They highlighted and circled the key words and numbers on the story page. I recorded in my log that now it was difficult to keep their attention.

We worked through two more stories, with students reading and retelling and answering the guiding questions on the Change checklist. Each example had a different pattern. In the second story, the first sentence told about the change and had to be mapped into the middle box. The third story had the change information in the last sentence. This complexity was intentional, and the students struggled each time to map correctly. They tried to map them all in the order the numbers appeared in the story, and they suggested adding each time as the operation.

Pat said, "I wish we could stay here and do math all day!"

He was happy and enjoying the process. He was not feeling confused, even though I knew there were some problems. I interpreted it this way: He had simplified the task in his mind and could be successful doing it his way. He had listened to the first explanation carefully. He thought he understood what to do, and he could do the mapping of the first type of problem. He did not do the critical thinking step that would identify the correct placement of the numbers in the diagram when the more complex language was used. The other students were the same. I knew more guided practice was needed. The students needed to be flexible about mapping the facts. The facts would not always be sequenced in the same order.

Collins and Carnine (1988, as cited in Carnine, 1997) advise that reteaching the original strategy is the preferred method of corrective feedback if the original strategy has been refined as a "best" approach. Jitendra and Hoff (1996) had refined the instructional language for teaching the Change diagram strategy and written a script for presenting it. In order to correct the students' errors, I reviewed the script so I could reteach it better. I did not find any errors in my presentation except in the fusing of the first two steps. When I presented the lesson again, I had the students check off the two questions in Step 1 without highlighting anything in the story. Then we organized the facts in the diagram as we responded to the questions in Step 2. Then the students could use their markers as we found specific facts and key words to highlight and map. In this way, I avoided my mistake when I re-taught and practiced the Change strategy.

During the next two lessons, I found it challenging to hold them back. They wanted to pick up markers and work ahead. They continued to make the same kinds of errors in the mapping. At that point, I decided to do something different.

I let them work individually, at their own pace, on three practice word problems. During this time, I circulated, observed, and provided individual instruction. After the practice problems were done, they had a four-problem quiz to do. I wanted to see what they could do on their own. I told them they could raise their hand for help. When I asked if there were any questions, I got some interesting responses.

Ben told about a dream he had had last night in which he knew "all the math". He knew the fourth grade and fifth grade and college math and he could do everything!

Andy was concerned that it was too many problems for him to finish. I said he could have all the time he needed. I read some of the problems to him because he is a slow reader. In the end, he did not finish. He had one left to do.

Pat worked confidently. He called out in a pleased tone to say, "I'm going fast."

Ben asked, "Is this right?" He showed me his diagram after he finished each problem. During the quiz, he insisted that he needed to go to the nurse because he might have a fever, like his sister. I sent him. He came right back, healthy, and did not ask again.

Terry quietly brought her diagrams to me before she erased them, and I encouraged her. I reminded her to borrow in subtraction problems.

Andy needed the extra time to finish his quiz.

Quiz 1: Andy – 75%, Ben – 100%, Pat – 88%, Terry - 100%

Two weeks after starting the strategy instruction, I had some concerns. I did not think they listened to the explanations when we mapped as a group. They did not engage in math discussions. When they heard my approval, they knew it was right. They did not check if the answer made sense.

They understood the parts of the Change diagram and how to write a math sentence from the map. During the quiz, the students appeared confident and their scores were acceptable. Quiz 2: Andy – 75%, Ben – 92%, Pat –25 *, Terry – 88% *Pat only completed one problem. He came to class upset and slept during the test. The one he did was correct.

Group Interview

I wanted to know how the students thought they were doing and how they felt about the problem solving lessons. I conducted an informal interview with the math group on the day after the quiz. I asked if they were tired of listening during math class. They all said they were not tired of listening. I asked them what they thought about doing the math problems on their own for the first time.

Pat said, "It was fun and it got faster. It was good. They can learn better."

Andy said, "I got slower."

Ben said, "Better, 'cause on tests I can get faster than working in a group. Sometimes I want to do the paper first and then explain it afterward, you know, check back."

Terry said, "Better. It is more quiet."

Although I was glad that they were mapping and solving better, I was concerned about our progress toward discourse about math. My assessment of their understanding was not as positive as their opinions of themselves. It was a good sign that they all said were not tired of listening. I read that to mean that they were not turned off to the strategy. They had developed some skill in using the diagram and the FOPS problem solving steps.

Problem Analysis

A few days after the group interview, I made a list of instructional goals and the problems they addressed:

1. Memorize FOPS – There are too many words on the checklist for them to read.
2. Fade the diagram so they can write it on their paper without the markers and plastic - it will give me a paper record and get rid of the distracting tools.
3. Provide contrasting problem types -To help them to identify Change problems by emphasizing the beginning, change, and ending in the problem.
4. Use familiar language from the Saxon curriculum to help them identify Change problems as *some, some went away* and *some, some more problems.*

Writing Explanations on a T- chart

A few weeks into the study, I taught the students to fade the diagram. Instead of using ovals and boxes, we just drew a line for beginning, change, and ending. They practiced creating their own diagram in this way. Then I taught the students to write an explanation of a Change problem using the T-chart. My goal was to give them repetition and practice with the problem solving steps to help them to memorize. I created a scaffolded T-chart format on a worksheet (see Figure 5). The page had a faded Change diagram with labels for beginning, change, and ending; a line for the number sentence; the word problem, double spaced; and a T-chart labeled Work and Explanation. Each student got a copy, and I had one for the overhead projector. I put the FOPS and Change checklist on the wall as a reference for our writing. We took turns following the FOPS steps to solve the problem.

The gray squirrel made a pile of nuts. It carried away 55 nuts up to its nest. Now, there are 38 nuts in the pile. How many nuts were in the pile at the beginning?

We read and retold, then underlined the question so we knew what we had to Find out. We circled the numbers in the problem, highlighted the change and labeled the beginning, change, and ending. Then we Organized the numbers by mapping on the diagram, Planned by writing the number sentence indicated by the mapping, and Solved by writing the math problem and solving it. We labeled the answer. I showed the students how to begin each part of the explanation by following the FOPS steps with these starting stems:

1. I had to find out…

2. First, I figured out that this is a Change problem because...
3. Next, I organized …
4. Then I planned to…
5. Finally, I solved …

As we created the explanation, I wrote it on the overhead and they wrote on their papers. The next day, I gave them a typed version of the explanation and a new problem for them to explain on their own as a test.

The students had 40 minutes to map the problem and complete the T-chart explanation. Terry and Pat finished in the first half hour. Andy and Ben needed the whole hour to finish the test. Andy, Pat, and Terry did their best to solve and explain the problem. Ben engaged in a string of avoidance behaviors, finally saying, "I don't like tests. Sometimes I like working in groups. I don't like the test. I don't like doing two parts and explaining."

In order to finish within the hour, I had to do the writing for him as he dictated his answer. I had to write for Andy also. He often works at a slow pace, distracted from his purpose.

Figure 5. *T-Chart Explanation - Scaffolded worksheet includes a "faded" map for change problems.*

Name________ Date________
Change Problems - Explanation POTD 1-A

? nuts	- 55 nuts	38 nuts
Beginning	Change	Ending

? nuts - 55 nuts = 38 nuts

The gray squirrel [redacted] (Beginning (? nuts)). It [redacted] (Change (- 55 nuts)) [redacted] up to its nest. [redacted] (Ending (= 38 nuts)) in the pile.
How many nuts were in the pile at the beginning?

WORK	EXPLANATION
55 nuts + 38 nuts 9 3 nuts Answer: 93 nuts	I needed to find out how many nuts were in the pile at the beginning. First, I figured out that this was a change story because it has a beginning, change, and ending. Next, I organized the information in the change diagram: ? nuts - 55 nuts = 38 nuts Then I decided to add 55 and 38 to get the number of nuts in the pile at the beginning because there were more nuts at the beginning. I had to add to get the BIG number. Finally, I wrote my math problem and solved it. I wrote the answer with the number and label.

Name [redacted] Andy Date 10 23 04
Change Problems - Explanation POTD 3 80%

297 box tops	+? boxtops	500 box tops
Beginning	Change	Ending

297 boxtops + ? bt = 500 boxtops

Mr. Frank[redacted]s class had 297 boxtops last Friday. By the end of the month, they need to have 500 boxtops to win. How many more boxtops do they need to collect?

WORK	EXPLANATION
500 - 297 203 boxtops	I nee ded to find out how many boxtops do they need. First I figured out this in a change story - Begn / mid / End Next I organized the information in the change diagram 297 bt + ? bt = 500 bt boxtops Then I decided to subtract because the big number. Finally I wrote my math problem and solved it. I wrote the answer with the number and label

Math Conversation

After the test on explanation, the students did one more T-chart for Change diagrams. This time the group did listen to each other and to me. They had different ideas about what to do. They changed their minds after listening to each other. Something had changed them over the last week. I had changed some things. I had made the work written and independent in an effort to focus their attention on what I wanted them to do. I had made the steps more explicit by making them write the thinking down in the explanations. When we began to write this explanation, we finally engaged in a group conversation.

I moved around the table, pausing between different students to support them as they tried to explain their ideas on how to map or how to explain. Again Ben had a lot of questions, in a run-on litany: "I thought we work in groups. I thought we use the Change chart. I don't understand the work and explanation part. I don't know how to start." This time I moved between Ben and Andy and directed them, thinking to keep them on task and moving at a reasonable rate. I was willing to take dictation from them for part of the task to move them along.

My Litany:

"First, Read and Retell.

"Follow your FOPS change checklist."

"Use the sample T-chart."

The problem was about a school-wide fundraiser our PTA is running. Each month the class that collects the most Box tops for Education wins a class treat for a prize.

Mr. Franklin's class had 297 box tops last Friday. By the end of the month, they need to have 500 box tops to win. How many more box tops do they need to collect?

I started to ask leading questions to engage them in going over what they had already done.

I said, "First, Read and retell. Follow your FOPS Change checklist."

Someone said, "It's about box tops."

I said, "Good. Change problems are all about one label: box tops."

They all wrote the label, box tops, in each part of the change diagram. They all had that right on their own.

I could see that Terry had the second statement mapped as the change part of the problem. It said, "They need to have 500 to win." The students needed to analyze the language to understand what was asked. Could they do it? Could I guide them? Could they attend and participate?"

I read the first sentence and asked, "Is it beginning, change, or ending?"

Mr. Franklin's class had 297 box tops last Friday.

Terry said, "Beginning."

The others agreed, so we all mapped it as beginning.

I asked. "What words tell you that it is in the past – at the beginning?"

Andy said, "Already had 297 box tops means in the beginning"

"Good explanation," I complimented.

"The second sentence is long and tricky. I'll read it aloud," I said.

'By the end of the month, they needed to have 500 box tops to win the contest."

I read it many times, emphasizing the key words: by the end of the month, they needed to have . . .

Terry still said, "It is the change number." Everyone agreed with her.

I said, "What words in the sentence tell you it is a change? There was some silence. I read it again. Then I asked, "Is that the change or the ending?"

Terry changed her answer, "It is ending." Everyone else still said change. I thought this was awesome. They were disagreeing and taking a stand!

I asked for explanations. Terry was looking confused. Ben was looking like he was just waiting. Pat was writing.

I asked again, "Who can explain their idea?" Then I leaned down and whispered to Terry that she was right. "Can you explain?"

She couldn't.

I asked them to look for words in the sentence that would tell where the 500 box tops belong in the Change diagram.

Terry stared at her paper and underlined- need to have.

I said, "Good. Everyone, listen. Need to have is a clue. Can you explain, Terry?"

"No," she shook her head.

I said, "Listen. How many do they need to have to win?"

They all said, "500."

I said: "Do they already have any?

"They all agreed, "Yes. 297."

"So," I asked, "Is 500 the change or the ending number?

Pat said, "Ending."

"Yes, Can you explain?"

Andy seemed to be thinking. Ben looked like he was waiting. Terry was not able to explain.

Pat explained, "They have to get 500 at the end to win."

I said, "Yes! 500 is the ending number that they need to have to win. They already have some, and they need some more to get to the goal of 500 box tops."

The word goal had made me think of a football analogy. I said, "It is like football. They have some, and they need to get more to reach the goal and win."

Ben got confused. He said, "There are no goal posts in the story."

I stopped him, saying, "Sorry, I mean that the end is like reaching the goal. Can you see where you should map the 500 box tops in the diagram?

Andy said, "The end."

"Yes! Now, the last sentence in the problem."

How many more box tops do they need to collect?

They had already put the question mark in the change part of the diagram. And they had started to subtract to find the missing small number. Except Terry. She thought she should add.

I said, "On the checklist under solve, it says: If the *big number* is given, subtract to find the other number."

Terry did not understand how to figure out if the *big number* was given. Clearly she did not understand that the ending number in this problem represented the total or *big number* for the story.

Three students wrote the subtraction problem with the larger number on the bottom. I redirected those three to put the larger number on top and remember to borrow when they subtract. Then, I showed Terry her number sentence:

297 box tops + ? box tops = 500 box tops.

I reminded her of number families. I said, "If you have a missing addend, you subtract." I tried another familiar explanation, "You work backwards to make the subtraction problem." And one more, "In adding, the biggest number is the *total* after you add."

She just could not understand it right now.

I said, "It's ok if you don't know this today. We will be practicing more and you will get it another day."

They finished by writing the explanation. Terry and Paul wrote quickly and followed the model to write their explanations. I supported Ben and Andy by doing some of their writing for them, as they dictated their sentences. After writing two sentences for each, they were able to finish the assignment on their own. Each student read part of his or her explanation.

Another Math Conversation - Terry and David

Although I was pleased with our first successful math conversation, Terry had not been successful in explaining or understanding the word problem we did together. Though she is very self-contained, she gets upset when she struggles. I had reassured her, but I was thrilled to have an opportunity to place her in the position of the expert explainer on this same day.

Terry and Andy had ten minutes of class time left. Andy needed to practice telling time and I knew that Terry had that skill. I set them up with a set of time flashcards. I told them to look at the top card and tell what time it says. Before checking, discuss your answer with your partner. When you both agree on the answer, turn it over to check. Put the correct cards on one pile and the mistakes on the other pile. I guided them through one conversation, modeling how to direct attention to the hour hand, then the minute hand. The next card came up: Andy said the time was 5:30. Terry disagreed.

I prompted her by saying, "Can you explain?" She could!

She said, "When it (the hour hand) is between two numbers, it is the lower number."

I agreed with her explanation. We looked to Andy to see if he understood.

He still had a question. He pointed to the card and said, in a questioning tone, "It is almost on the 5."

I used a note card to follow the line of the hour hand toward the five. It pointed to a spot just before the five. "Do you see that?" I asked, "It is not after 5:00, but it is very close to 5:00. So, It is still before 5:00."

I told them to practice together until it was time to go. "The drawings can be confusing, so talk together until you agree, then check. Keep the stacks so I can see how you did as a team."

I heard them talk and question each other. I was thrilled because Terry was an expert at this and could explain with confidence. Andy was getting needed practice. Plus he was good at asking when he was confused, which made the dialogue meaningful practice, not fake. They had missed only one.

I told them, "Awesome job! I'm hoping you can talk to each other and decide about word problems the way you decided about the time on the clocks. Then I would know that you really understand word problems."

I thought that it was great for Terry to have success as a person who explains. Earlier, she had struggled and given up when she had a chance to explain her idea. This day's work was the turning point in my effort towards discourse about math.

The Group Schema Diagram

I had planned to teach to mastery of 100% correct on two consecutive daily quizzes before introducing the next schema diagram. At this point in my study, the students had scores for three quizzes, with scores of: Andy – 75%, 75%, and 92%; Ben – 100%, 92%, 92%; Terry – 100%, 88%, 83%; and Pat – 88%, 25%*, (slept), 83%. I decided to teach a contrasting type of problem. I hoped it would give them a context for understanding what was meant by the "change" in a Change problem.

Rosenshine and Meister (1992) pointed out the importance of controlling the difficulty of learning a new cognitive strategy. They gave examples of how other researchers anticipated and discussed student errors to help their students recognize the mistakes in the examples. Providing samples of incorrect responses helped to clarify what makes a response correct. I decided to teach the Group schema diagram to give my students a contrasting model, hoping that the contrast would help define both of the models: Change and Group.

Semantics and Understanding

First, I had to teach the semantic elements that the Group schema represents. I began by stating that small groups could be part of a larger group. This large group is the whole group. I asked if anyone remembered the lesson when we folded square pieces of construction paper to show parts of the whole square. They all raised their hands to show that they remembered. I said that another kind of whole group is our math class. Two small groups that are part of the class could be students and teachers. I asked if there were any questions and they all shook their heads.

"Okay then," I said, "I will read some lists of three words. You tell me which word is the large group and which words name the small groups. The first list is: oranges, apples, fruit. What is the large group?"

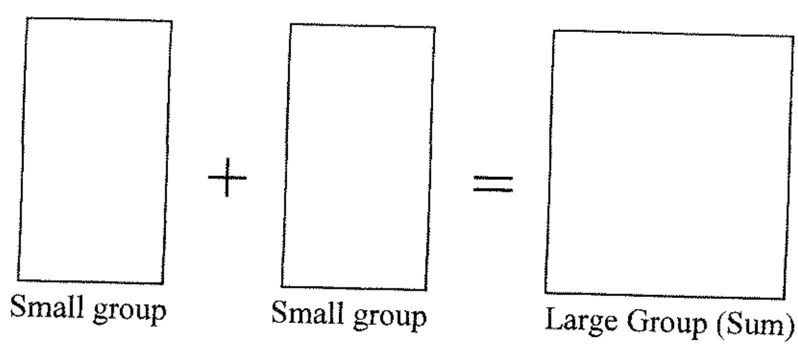
Some of the students called out, "Fruit." Others did not answer.

I said, "Fruit is the large group. Right. The small groups are kinds of fruit. Oranges and apples are the small groups because they are part of a whole group called Fruit.

"Okay, Let's see if you can figure these out. I read five more lists and got their correct responses.

Then I displayed the group diagram on the white board

Figure 6. *Group Diagram*



I presented the first story:

Anne has three apples and 2 oranges. She has 5 pieces of fruit.

I modeled the FOPS problem steps by following the Group Story Checklist.

Figure 7. *Group Story Checklist*

GROUP STORY CHECKLIST

Step 1. Find the problem type.

- [ ] Did I read and retell the story?
- [ ] Did I ask if it is a group problem? (Did I look to see if two or more small groups combine to make up a large group?)

Step 2. Organize the information using the group diagram.

- [ ] Did I underline the large group and small groups and write in group names in the diagram?
- [ ] Did I circle numbers for the groups and write in numbers for groups in the diagram?

Step 1: "To Find out if it is a group problem, I retell it in my own words: The story is about a girl who has 5 pieces of fruit. She has three apples and two oranges." I continued, "The next thing on the checklist says to look for two or more small groups that combine to make up a large group. What do you think the large group is?"

Ben answered, "Pieces of fruit."

I said, "Okay. Then do we have small groups that are kinds of fruit?"

The class answered aloud, "Yes, apples and oranges."

I said, then, we can check the box that it is a Group problem. Now, we will organize the information in the diagram.

Step 2: Organize, "Let's underline the large groups and the small groups and write the labels in the diagram," I said, as I used a marker on the transparency to underline fruit and oranges and apples. I went to the board. "The small groups are apples and oranges so I will write apples in one small group box and oranges in the other small group box. The large group is fruit, so that is the label I will write for the large group box. How is that different from the Change diagram?"

The students knew that Change problems have only one label for all the at once.

We read, underlined, labeled, mapped the numbers, wrote the number sentences, and checked three stories using the overhead projector. The key decision the students had to make was to determine what label represented the large group.

The second story had three small groups so I showed them that they could draw extra boxes in the diagram for more small groups. In the third problem, there was an extra number that was not needed. We talked about how to cross off extra information, and then use the diagram to map the facts that matched the labels. They were attentive and involved.

In my log I questioned what made this teaching session more productive than the early teaching sessions on Change diagrams. I thought they understood the diagram and the strategy steps better. Perhaps it was easier to understand because the strategy steps were familiar and only the diagram was new. The Group diagram was a new twist to a now familiar process of following the FOPS checklist and mapping. It was new enough to hold their interest and familiar enough to be within their zone of proximal development (Vygotsky, 1978). Vygotsky taught that "a person can imitate only what is within her developmental level" (p. 88). I hoped the students gained confidence as they successfully showed their understanding of the semantic concept of large groups and small groups. I was encouraged to keep pushing them to think carefully, to keep using the metacognitive guides, and to keep promoting discourse about math problems.

Modeling Divergent Thinking

The lesson moved on to the guided practice. Each student received a page with five stories to map, all using our names. We followed the FOPS problem solving steps on the Group checklist poster on the white board and we shared the Group diagram poster. After we mapped the facts, each student wrote the math sentence on their paper. We worked together, with the students taking turns in the teacher role by reading, retelling, labeling, and mapping the numbers, and checking as I called on them for a turn.

Two of the problems were about the number of people in a family, and they could be mapped in more than one way. One said, "Andy had 2 sisters and 1 brother. Counting him, there are 4 kids in his family.

Some kids wanted to map two groups: 2 sisters + 2 brothers = 4 kids

The other way is to map three groups: 1 self + 1 brother + 2 sisters = 4 kids

The students understood the two ways to map and eagerly participated in the conversation about it. I collected their papers with their math sentences on them. Of the four students, two students wrote the family problems using three groups and the other two students wrote the family problems using two groups. They took shortcuts in labeling also, using g for girls or 4g for fourth graders. This decreased the writing demands for my two reluctant writers.

What is the Big Number?

The next day, I presented one Group problem on the overhead projector. The problem was complicated because the large group number was given, so we had to work backward and subtract to solve. I used the problem and the script that I had received from the workshop on using the schema diagrams. The problem:

There are about 75 different flavors of ice cream. Baskin Robbins has 35 flavors. How many flavors of ice cream are not at Baskin Robbins?

After we worked through the mapping and solving, I could see that there was some confusion with identifying and labeling the small groups and the big group. Also, I could see that the students did not clearly understand the idea of the *big number* – the *whole*, the *total*, and the *sum*- no matter what I called it. To keep them focused and thinking, I wanted to get away from the teacher-guided setting.

I told them we would be working with partners to map and solving some group problems.

Levels of Peer Collaboration

Andy said, "Can we work with different people? I want to work with Terry."

Ben said, "I want to work with Pat!"

Since no one objected, I said, "OK." Pat left the room to go to the bathroom. While he was gone, I complimented Ben on his making the partner switch work by eagerly wanting Pat for a partner.

Pat has been sad a lot during the last two weeks. He said he was sad because of friendship trouble with his classmates. He is part of his own problem. It seems that he is trying to get attention by acting sad and forlorn. His sadness is real, though, and rooted in more than peer relationships. But this math group has been patient and supportive of Pat. Ben's actions today in his enthusiastic embracing of Pat as his partner were especially supportive. I was proud of them all. Donahue (2002) writes, "Perhaps at no time in history have so many diverse theoretical perspectives converged on the importance of positive social interaction to human development. Even medical models now acknowledge the benefits of peer support to physical and emotional well-being" (p. 240). The way we feel about ourselves affects what we can accomplish. In a supportive atmosphere people feel it is safer to take chances. I wondered if this was a step in the right direction, or a sign that we were already there in creating a safe place to problem solve.

The students collected their folders, markers, and partners and found a spot where they wanted to work. They wanted to sit on the rug, so I set up a money activity on the table, in case one of the groups finished early. I listened and observed from a distance to minimize my interference in their discourse. They had seven word problems to solve. The partners followed the steps and mapped with the markers on the Group diagram in the plastic sleeve. Then they did the math problem and recorded the number sentence and the answer on the paper copy. Quiz 5: Pat- 58%, Ben - 71%, Andy - 71%, Terry -71%

Pat's score was the lowest. His math sentences were wrong only twice, but he made five errors in computation. The other students made only one error in computation each. Pat did not always have the same answer as his partner, Ben. They got along and mapped and talked, but if they disagreed, they each wrote down their own answer. Pat's partner Ben was confused by the verb "sold" in the first question. Ben made this error, although his partner, Pat, had the correct answer. In Change problems, the verb indicates the operation, add or subtract – and "sold" can indicate subtraction, when taken out of context. The sentence said: They sold 14 goldfish and 11angelfish. The meaning of the whole sentence had to be considered in order to decide to add. I wondered why some partners had different answers than their buddy.

I observed Pat with different partners to see why some of his answers were not the same as his partner's answers. I saw that he would share, keep up, and use his partner's ideas to map and solve his problems. But he did not check to see if his answer matched his partners. After they decided what to do, he did his own writing and solving. The step of checking the answer was not being used.

Elksnin (1997) described four levels in relationships that lead to working collaboratively with a peer. The lowest level is co-activity, which resembles children's parallel play. Space is shared, but nothing else. In the second level of co-operation, the peers jointly establish general goals, and share ideas on how the work can be done, but each person is responsible for his or her own work. The third level is co-ordination in which the two develop a sense of cohesion. They share ideas and strategies when there is a need, but still do their own work. The highest level is collaboration. In this situation, the peers share responsibility and ownership for all the work that occurs. It takes time to develop the relationships and the skills to work collaboratively, and co-operation is a step along the way. I was encouraged by the way the students cooperated with each other. I was satisfied with the >70% accuracy rate for this session.

The math students had another practice session a few days later, solving three word problems with a partner. They worked with the same partners: Pat with Ben, and Terry with Andy. Terry and Andy recorded the same-shared answers, both scoring 78%. Pat and Ben again worked together, but recorded their own answers, scoring 99% and 78%.

The next challenge would be to present both types of problems and the first decision would be to identify which diagram to use – Change or Group.

Change or Group

For the next two weeks we puzzled together over mixed sets of word problems, deciding which diagram to use to map and then solve them. I saw that they could get the right number sentences even if they could not choose the right diagram. When I said we would be working with partners, Ben said, "I want to work with Andy." So they went back to original partners with no other conversation. As the partners worked, I walked around, observing and guiding.

I was pleased with their attitudes, cooperation, attention, and flexibility. No one shut down when they were confused. When someone else had an idea, they agreed or disagreed calmly. They took themselves seriously and kept working towards a representation of the problem and a solution. Their written work was neat and complete. They knew the FOPS steps and followed the strategy sequence without the checklist. They had a strategy for how to approach the task of thinking about math word problems. They used the steps: Find, Organize, Plan, and Solve.

From notes in my field log I wrote a pastiche to capture the flavor of the simultaneous math conversations the students engaged in at this midpoint of the research study. They did not have the checklists in front of them, only the problems and the diagrams.

Figure 8. *Pastiche of a Math Conversation*

Money for the Movies

Terry and **PAT**, **Ben** and Andy, and the Teacher

Ben led his group:

You can read, Andy.

He is not the best at reading.

I'll help him.

Good idea.

A child's ticket to the movies costs $5.50. An adult's ticket costs $7.50. If Sally goes with her mom and dad to a movie, how much will the tickets cost? *(Change or Group)*

How about I explain if it is group? The small group is $5.50 and the other one is $7.50 and we don't know what is in the big one. Now you tell if it is change.

(pause)

What do you think about change?

It is group.

If we write "tickets" for this small group, what about this one?

Tickets for the second group.

Okay, adult tickets.

At the same time the other group – *Terry* and **PAT** said:

Let's read it.

A child's ticket to the movies costs $5.50. An adult's ticket costs $7.50.

If Sally goes with her mom and dad to a movie, how much will the tickets cost? (Change or Group)

I THINK IT IS A GROUP PROBLEM.

The small group is child tickets.

OKAY

This group. . . . tickets?

ADULT TICKETS.

Big group- -tickets for the movies?

YEAH. YEAH.

I think it's plus.

PLUS, MINUS, PLUS, MINUS, PLUS, MINUS, PLUS...

Stop, Pat.

Let's see what you have. Hmmm.

You labeled the boxes correctly. Good.

Who went to the movies?

TWO PEOPLE. THE MOM DIDN'T GO, DID SHE?

Read the sentence. If Sally goes with her mom and dad to a movie, how much will the tickets cost? Who goes?

MOM, DAD, AND SALLY

Mom, Dad, and Sally

Where will we map that, adults' tickets or child's tickets?

ADULTS' TICKETS

(Now, both groups were listening and involved.)

We have to draw another box.

Right

(Each drew another box on the group diagram and labeled it.)

What numbers go in the boxes?

You can put both adults in one box. Let's see, $7.50 + $7.50 = $14.00.

You could do it that way. But $14.00 is not right. It is a tricky one to add in your head.

HOW CAN WE DO THAT?

Yeah, how can we do that?

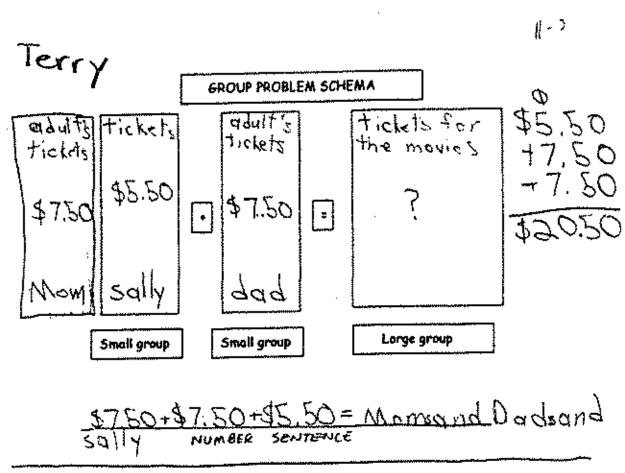
You just add all the numbers in a long column.

OH, YEAH. I CAN DO THAT.

Me too!

When I saw our conversation in this format, I worried that the teacher still had too prominent a role in guiding the thinking, but I was happy to see the students using the FOPS steps. They demonstrated that they knew this step-by-step process for solving word problems.

Figure 9: *Student Map*



Error Analysis

I wanted to know what problems the students were still having, so I analyzed their written work to map the problem areas. When were the students confused? What errors did they have in their work? I used the FOPS checklist to sort the errors. I used tally marks to show mistakes. The tally marks (1) are from partner work on a four-problem quiz done at the midpoint of the study.

Table 3: FOPS Error Analysis 1 – *Errors by category as student pairs solve Change or Group problems*

Name	Find	Organize	Plan	Solve
Ben		11		
Andy		11		
Terry	1	111		
Pat	1	11		1

Note – tally marks = 1 error

The problems were clustered in the Organize step. This indicated to me that the students were struggling to represent the word problems using the diagrams. The students needed help to label the large group in Group problems when the sentence had the large group information first, instead of last. In the diagram the large group is mapped in the last box. The students got confused when the information in the story is in a different order than the mapping requires. When they worked with a partner, they would often correct each other, or recognize their confusion and ask me for help. When they worked on their own, different students did different things. An underlying problem could be confusion in identifying the total or *big number* in problems.

Table 4: FOPS Error Analysis 2 – *Students error patterns on three word problem quizzes*

Name	Find	Organize	Plan	Solve
Ben	2 2 3	1 1 2 2 2	2 2	2
Andy	2	1 1 2 3	3	3
Terry	1 2 2	1 1 1 2 2	2 2	2
Pat	1 2	1 1 2 3	3	1 2

Note- tally marks show errors. 1 (Quiz 9) = pairs at midpoint of study, 2 (Quiz 14) = independent at end of study, 3 (quiz 15) = pairs at end of study

In Table 4, I added tally marks (2s) for the errors students made while working independently on a 4-problem quiz that was done near the end of the study. I saw that scores went down when students worked on their own. Andy helped me check the trustworthiness of this view during his individual interview. He asked me to read the problem for him after he had tried it on his own. After I read it to him, he could tell me that the numbers had to be 'flipped around' in the diagram. I asked him what helped him figure it out. He explained, "When the teacher read it, I didn't get the words mixed up." These clues point to the semantic analysis of the language in the word problems as a stumbling block for the students.

I wanted to find out how their scores while working with a partner compared to their scores for independent work. So the next day, I paired them up randomly to work on 10 problems together. Terry and Ben were partners and Andy and Pat were partners. I recorded tally marks (3s) for the errors they made. The partners did very well with the word problems, scoring: Terry – 100%, Ben – 90%, Pat – 87%, Andy – 93%.

The students' conversations sounded like this near the end of the study:

Figure 10. *Conversation Plays*

<u>Andy and Pat #2</u>

Andy: Mike has 6 fish. His friend Juan gave him 4 more. How many total fish does Mike have now?

(Pat drew three boxes, indicating a Group problem)

Andy: You only need two boxes.

(He drew 2 boxes and wrote in: 6 fish + 4 friend fish = 10 fish)

<u>Andy and Pat #3</u>

Andy: Melita bought shirts today. She bought 3 shirts at Wal-Mart, 2 shirts at K-mart, and 1 shirt at Kohl's. How many shirts did she buy in all?

Pat: Add – Change.

Andy: No! Group. You have to add. It says - in all.

Pat: I agree – Group.

Andy: This one needs three boxes, equals four in all.

Andy: Label "shirts in all" at the end.

Ben and Terry #2

Terry: Mike has 6 fish. His friend Juan gave him 4 more. How many total fish does Mike have now?

Ben: (Retold it by rereading it.) It's group.

Terry: It doesn't need three boxes.

Ben drew boxes and labeled: (6 fish + 4 more fish = 10 all the fish)

Terry drew boxes and labeled: (6 fish + 4 more fish = 10 total fish)

Ben and Terry #8

Terry: Terry ate a piece of cake last night after dinner. She ate another piece before she went to bed. This morning she ate another piece for breakfast. How many pieces did she eat in all?

Ben: Change – minus it.

Terry: Plus

Ben: Oh, Group with I think, Change.

Terry: Group

Ben: Ok, let's do group this time.

Ben: Three boxes.

The students corrected each other and helped whoever was not thinking clearly. It is interesting that different students shone on different days. No one stands out consistently as making more progress than anyone else. In solving #2, both groups used the Group diagram to solve it. And both correctly solved the problem by adding. It could also be mapped with the Change diagram. Ben did not make sense in the discussion of #8, but his partner got him back on track. On other days, Ben had been a leader in solving correctly. It is a characteristic of learning disabled students to be inconsistent, to have days when they can access their memory and days when they cannot (Dornbush and Pruitt, 1993).

Money Math

Near the end of my study, two new students joined our math class. They entered the class two weeks apart in the weeks between Thanksgiving and Christmas. I decided to reteach a lesson using money in the Change diagram to teach the new students about the mapping strategy we had been learning. In this lesson, every word problem had the same language and mapped in the same way in the Change diagram. I wanted the students to practice mapping facts into a diagram in a different order than they appeared in the word problem. I wrote the word problem on the board with blanks to fill-in to change the facts:

_____________ bought a toy for $________.

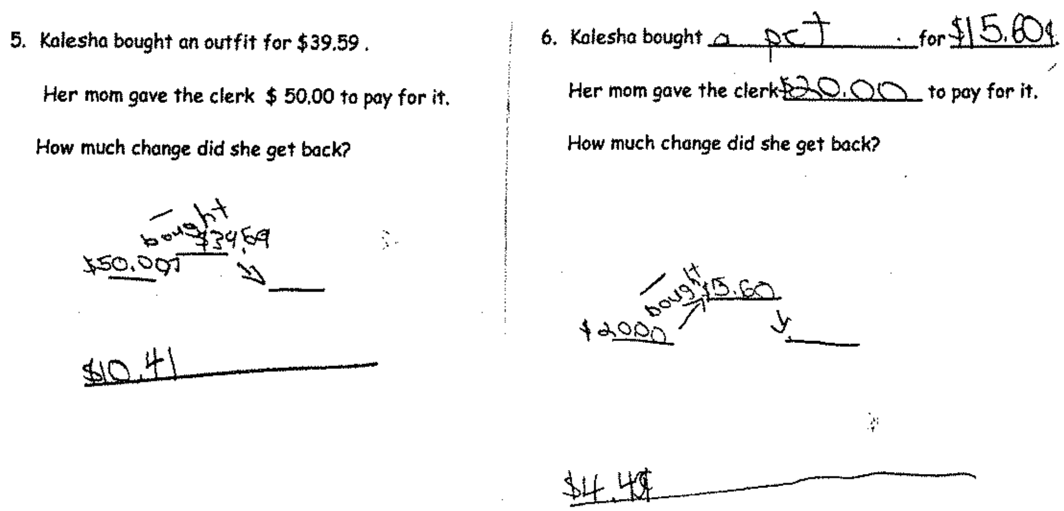
His / Her mom gave the clerk $________ to pay for it.

How much change did he/she get back?

Each student created a word problem by starting with his/her name, choosing a price for the toy, then choosing an amount of money to pay for it. We talked about making sure they had enough money to pay for the item. We followed the FOPS steps and mapped on the white board. We worked through five problems together, one for each student. The students used calculators to figure out their answers. They checked with each other to see if they had the same answer. If there were different answers, they redid the math on the calculator. Then they counted out the change due from their own play money cash drawer. After the five problems, I passed out a six-problem quiz with the same type of problems. Three of the problems had money amounts given, and in the other three they chose the numbers to use. They had to map with the Change diagram and show the computation problem. They could use the calculator to get the answer. The students in the study scored very well:

Andy = 83%, Ben = 100%, Terry=100%, and Pat= 83%

Figure 11. *Student Work – Money Problems*


5. Kalesha bought an outfit for $39.59.

Her mom gave the clerk $ 50.00 to pay for it.

How much change did she get back?

$50.00 bought $39.59

$10.41

6. Kalesha bought a pct for $15.60.

Her mom gave the clerk $20.00 to pay for it.

How much change did she get back?

$20.00 bought $15.60

$4.40

The students really enjoyed this lesson and it was a pleasant way to welcome a new student and introduce problem solving. They liked using the calculators and cash drawers, and making up their own numbers. They helped each other and compared their answers to check. This working discourse was the atmosphere I wanted for math problem solving. They were confident that they knew how to solve the problems.

This started a series of lessons on shopping. In the next lesson I put the two types of problems together to introduce two-step problems. I introduced the lesson by evoking a shopping trip. I asked one of the girls which store she liked at Carnival Mall. She mentioned Fling's, a store that has accessories for girls. I explained that sometimes when we shop, we buy more than one item at the store. At Fling's you might buy a purse, shoes, and a shirt. When you take the items to the clerk, the clerk adds up the prices of all your items to get the *total* amount of money that you are spending. This would be a Group problem.

I showed them how to map it in a Group diagram. Terry noticed that I'd needed another box in the diagram, which I drew onto the Group diagram poster. They already knew how to use the Group diagram, so we mapped and added to get the *big number* or *total* price.

Then I wrote the second part of the problem on the board. This was in the same language as the Change problems we had just practiced the other day:

Terry spent __________ at Fling's.

Her mother gave the clerk $40.00.

How much change did she get back?

I asked the class, "How much money did Terry spend at Fling's?

They told me, "$35.00." I wrote it in the word problem. We mapped it in a Change diagram and calculated the change with calculators. Then I assigned two problems to work on with a partner. They all got 100%. They worked on this type of two-step problems, with partners, for two days, and then they did two problems on their own. The independent quiz scores indicated which students were better independent workers:

Ben - 100%, Terry - 100%, Andy – 50%, and Pat – 33%

We would continue to practice solving word problems for the rest of the year, but my study was coming to an end. During the next week, I assigned independent work and interviewed the students individually. During the interviews, I asked the student to solve three word problems and answer survey questions about their strategies and attitudes (see Appendix K).

Math Center

For the two last Math Centers of the study, two new students were included in the groups. I chose pairs by picking names. Two students had partners who had not been in class to learn the schema diagram strategy. Now that there were six students in the class, I designated three areas in the room for pairs to work. One center was structured around identifying the *big number* in math problems. Each student cut out 20 paper fortune cookies. I directed them to use the cookies to act out the story problems. They had to write the number sentences and their answer. I said they could use the diagrams, but it was their choice.

I defined *total* as the number that means everything all together. We modeled the first story together and identified the *total*. Then we circled the number and label in the number sentence that represented the *total*.

> Jason has 4 cookies. A friend gives him 2 more cookies. How many cookies does he have? 4 cookies + 2 cookies = 6 cookies

They worked with their partner to create a model of the problem with the cookies, create the number sentence, and label the *total*. Their scores were: Ben – 87%, Terry – 87%, Andy – 75%, Pat – 63%

Everyone circled the *total* correctly in addition problems, but in subtraction they still circled the number after the equal sign as the total. No one chose to use the diagrams to map the problems. When I asked them why they did not use the diagrams, they said the problems were easy and they did not need them.

For the last Math Center during the study, I chose six cards from the puzzle box: two easy, two on level, and two challenging. I read the cards to the class. Then, with their input, I gathered the materials for two problems at each of the three designated centers. I had copied the problems on papers so I could collect their answers and evaluate them easily for the field log. The partners were randomly chosen and included the two new students. I let one group choose where to start, then another group chose, and the last group took the remaining spot. After fifteen minutes, they switched areas and problems.

The partners struggled with the first problems they worked on. After they got more comfortable with their partners, they all started to work together more carefully on the problems. I was surprised when they all struggled with the money problems. They found it difficult to add change past one dollar. After the first pair struggled, we figured out that the calculators could be helpful. We added calculators to the tools in that center. One money problem was a two-step problem, and no group did it correctly without help. They did not rely on the diagrams, just went after the solution in their own way. The rich problems presented some challenge by asking them to add three hours instead of 1 hour to the time; to add a lot of coins, first 18 coins, then 37 coins; and in the two-step problem, they needed to add and subtract to get the answer. They did not get the details right, even with their partners. The challenges were not met.

The things they did correctly were components of the strategy. They did read and retell the problems. They did write labels on the problems or with their answers, but in their attempts to solve problems, they were not precise.

Scores: Andy – 50%, Terry – 75%, Ben – 50%, Pat – 50%

In the surveys and interviews, the students said this about problem solving as we discussed the problem they were doing or word problems in general:

Figure 12. *Pastiche – Knowledge and Confusion*

I knew . . .

To add to get the total

It was group because it had three numbers

I could retell it if I didn't understand it

I should read the problem to find how much you need to get

I understood the group and change

Group was plus

You subtracted the number she paid and added the number she bought

You set it up so you can check

You could use a calculator to check

When the teacher read it, I didn't get mixed up

I got confused . . .

When I read it myself

When I had to subtract

When I had to pick change or group sometimes

When I worked by myself

When there were two steps

When I had to count a lot of coins

Throughout my study, the teacher needed to be involved in the students' problem solving dialogue. I wrote this poem from conversations in my field log to describe the dialogue between the student pairs and the teacher:

Figure 13. *Poem: Problem Solving Dialogue*

What do you think?

I think you should subtract.

Change is subtract.

Do we have a big number?

What is the big number?

Oh yeah, that is what the total is.

They had 24, but they need to get 30.

I think you add.

No, you take away.

Let's use the Change diagram

What is the ending?

They need 30 at the end.

Ok. What is the beginning?

They have 24.

We need to find the change.

Do we add or

Subtract?

I know.

I don't know.

I think I know.

Draw the map.

I will __________.

Does it make sense? YES or NO

ANALYSIS

MacLean and Mohr (1999) wrote that teacher-researchers do not collect data for long without stopping to reflect and analyze. In my field log, I wrote observer comments as part of my daily notes to reflect on the students' response to the strategies we were learning. "Analytic memos are also a way to record methodological dilemmas – what to try next, given that the things already tried do not seem to be working, and hunches about what is happening" (Arhar, Holly, & Kasten, 2001, p. 187). I wrote analytic memos to record the progress and problems the students had with the schema diagram strategy and partner dialogue. I also used analytic memos to plan what I would do next.

After a few weeks of collecting data, I began to code my field log. Coding is the application of a descriptive label to pieces of data. The codes are a way of organizing the data into categories (Arhar, Holly, & Kasten, 2001). For example, I wrote the code *EXPLAIN - S* in the margin if I described a student involved in explaining part of a math problem.

Near the end of my study, I grouped my codes to create bins of similar categories. The collected codes suggested titles for each bin. As I organized my codes under the new headings, I began to see patterns and relationships in the data. I wrote theme statements that described the students' experiences and my experience. This process of taking the data apart and then putting it back together helps to "create connections and integrate things in your head" (Arhar, Holly, &

Kasten, 2001, p. 197). My theme statements guided my explanation of what I learned.

To verify my findings, I triangulated my data sources by using multiple sources of data, multiple points of view, and multiple methods (Arhar, Holly, & Kasten, 2001). I recorded problem-solving conversations between the student pairs and group conversations in the teacher-directed lessons. I also recorded spontaneous comments from the students about math class and problem solving. The students completed a survey before the study began in September. After the first few lessons on the schema strategy, I conducted an informal group interview to ask questions. The students answered the same survey questions during individual interviews in December, as my data collection ended.

Arhar, Holly, and Kasten (2001) explain that "critical researchers argue that the more we reduce power differences so that participants have an equal say with researchers, the closer we come to making valid statements" (p. 206).

I asked frequently for the opinions of the students in my study. I also asked the members of my researcher group for their interpretation of the data in my study. By seeking broad consent within a group, teacher researchers develop and test for "truth" (Arhar, Holly, & Kasten, 2001).

I portrayed the students' problem solving conversations in the form of a pastiche. This literary form "directs the reader's attention to multiple realities" (Ely, Vinz, Downing, & Anzul, 1997, p. 97). I used the pastiche to create a "graphic textual framework" to show the actions of all the participants at once. "Pastiche assumes that the pieces that make up the whole communicate particular messages, above and beyond the parts." (Ely, et al., 1997, p. 97) The meaning of the pastiche is to be constructed by the reader, rather than interpreted by the narrator (Ely et al., 1997).

I also analyzed my log by looking for metaphors. "A metaphor is a form of symbolic language in which one thing is used to describe another." (Arhar, Holly, & Kasten, 1999, p. 63) I listed the metaphors I found in my field log and wrote a reflective memo to explore their meanings. As I read and reread my log in the course of writing my story, an image began to emerge that helped to describe my class's experience with mathematical word problems. Creating these forms helped me interpret the experiences of my students as they engaged in dialogue to solve word problems.

I collected copies of the students' written work. I graded it by awarding points for organizing, planning, and solving for each problem. I used these points to analyze the students' progress and confusions in using the steps in the schema diagram strategy. I planned my teaching to clarify areas of confusion. Later, I used these data to create Table 3 and Table 4 to display the students' error patterns during the study.

I recorded each student's scores on schema strategy quizzes by recording the percentage correct and displayed the data in Table 5. I compared their scores to descriptive information about each quiz in Table 6 to draw conclusions about what they understood. I also created Table 7 to examine the students' scores on rich word problems from the math center. By examining the scores and the tasks, I could make judgments about what the students learned.

Table 8 displays data from the student surveys and the problem solving interviews. It compares their opinions from the beginning and end of the study. This helped me to judge the students response to the intervention.

FINDINGS

I found that strategy use helped students to talk about problems in an organized way, with common terms. The schema diagram strategy was an important part of establishing this common language. The four problem solving steps, and the expanded problem checklists that included metacognitive prompts for each step, became the framework for our problem solving conversations. The students learned to use the steps and they structured their math dialogue with each other in the sequence of the steps. Students this age like to have fair turns. Reading and retelling were the first steps in the problem solving plan. When they worked in pairs, they took turns, one reading and the other retelling, which served as an easy transition into the activity. The students shared a common understanding of the sequence of steps to follow to complete the problem. Partners exchanged opinions and then chose which diagram to use. Their written work included maps, labels, and complete number sentences. The organization of the materials helped to facilitate problem solving with a partner by giving them a clear sequence of steps to follow to complete the task. The task was defined clearly from beginning to end.

The establishment of respectful discourse patterns took some time. At first the students did not seem to be listening when others explained their reasoning. They did not give me their undivided attention as I thought-aloud through the metacognitive strategy steps for Change diagrams during the first two weeks.

They were busy with their materials, mapping and solving ahead of me. At first, I described their behavior as rushing. After we mapped the first problem, they thought they knew the pattern and tried to map each question in the same order. Later I came to see that rushing did not capture the whole reason for their behavior.

I found out that they felt successful when they got done fast. Pat said, "I'm going fast!" Ben said, "I like it because I can get faster." Andy lamented, "I got slower." The others perceived Terry as the best at math because she was always done first. Perhaps working ahead and working quickly indicated that they felt confident about what they had to do. But, when the language of the word problems was complex, they made mistakes in organizing the information. I noticed that some of my practices encouraged the student to rush through the word problems. By giving them a stack of puzzle cards during Math Centers or a page of word problems to solve, I communicated that they should try to "get through" the work. I believe, now, that carefully selecting only one problem to solve in a session would communicate a clearer message to take more time to think and discuss.

I found that they needed to be slowed down so they would correctly interpret the semantic clue words in the word problems. These word clues were like guideposts, indicating relationships between the number facts in the story. They needed to look for the verb in Change problems, but focus on the words that defined a large group and its parts in Group problems. When I guided them through the task of analyzing the clues in the words, they did not at first follow along with me. They were already writing their answers down. When they mapped some problems on their own, they did not distinguish the semantic clues in the sentences. My response to this "rushing past the guideposts" was to search for a way to slow them down and to focus their attention on the important words.

When the students had to write the T-chart explanations, it slowed them down and forced them to go through the metacognitive questions for each step. After that slower pace for two days, the third T-chart resulted in our first dialogic math conversation. We were all thinking and talking about the same thing at the same time. I was guiding, and they were helping to search for clues. From then on, they engaged with their partners, and as a class, in a structured dialogue to analyze the semantic clues in word problems in order to map and solve.

The schema diagrams helped the students begin to engage in dialogue because they provided a focus for everyone's attention, both visually and cognitively. The concepts of the beginning, change, and ending were ideas to think about and places to look in the diagram. I could point to places in the diagram as we talked about the relationships between the facts in the story. Consistent use of the FOPS strategy steps created an "instructional frame" (Anderson, in Cazden, 1997) that allowed the class to proceed automatically through the steps for solving word problems, and freed the students to engage in discourse about the meaning of the word problem stories.

I found that certain steps in the process were more difficult for the students than others. I constructed an error analysis chart to compare partner work from the middle of the study: (1); with independent work done at the end of the study: (2); and with partner work done at the end of the study: (3).

Table 4. *FOPS Error Analysis 2 - Students error patterns on three word problem quizzes.*

Strategy step:	*Find*	*Organize*	*Plan*	*Solve*
Ben	2 2 3	1 1 2 2 2	2 2	2
Andy	2	1 1 2 3	3	3
Terry	1 2 2	1 1 1 2 2	2 2	2
Pat	1 2	1 1 2 3	3	1 2

Note- tally marks indicate errors 1 = pairs at midpoint of study (Quiz 9), 2 = independent at end of study (Quiz 14), 3 = pairs at end of study (Quiz 15).

In step one: Find, students had to decide which problem-map to use. When they had to choose between Change and Group maps during the study, the students struggled with this categorizing task.

In step two: Organize, the students had to map the information into the diagram. The students struggled to translate the language of the word problem into the categories represented on the maps. This step was the central meaning-making element of the strategy, and it was not mastered during the study. Mapping words and numbers from the problem into the diagram created a new skill for them to learn. Also, previously learned directions did not match the new directions. In the past, they had underlined the question sentence in the story and placed brackets around important information. Now, they highlighted labels and important information in every sentence, and circled numbers. Hohn and Frey (2002) encountered the same result in their study of a step-by-step process for problem solving. The fourth grade students in their study did not make progress in using the SOLVED strategy. They postulated that these students "struggled to integrate a new approach with procedures that were previously taught or developed through their own efforts" (p. 7). The other students in Hohn and Frey's study did make progress. Possibly, older students in the study were more flexible, and younger students had no previous habits to replace.

I tried to mitigate the confusion caused by layering this new approach onto previous approaches. I used highlighters to highlight, instead of underline, the new categories of information. I added familiar language to describe the Change situation, as *some, some more* stories and *some, some went away* stories. I added discourse as a tool. When students worked in pairs, they were more successful at this step. In my error analysis chart, students working independently made sixteen errors and students working in pairs made two errors in this step.

In step three: Plan, the students had to decide whether to add or subtract. When I evaluated error patterns at the end of the study, there were only six errors in this step. The students were given a rule to lead their decision: If the Big number is missing, add. If the Big number is given, subtract. This rule did not make sense to the students when they heard it. I discovered through observation and conversation, that the concept of the total (Big number) was not clear to the students. They had procedural knowledge of number families, where the total and two small groups can create four related number sentences, but the relationship between the total and the small groups had not yet crystallized for them. I hoped that the graphic representations of the schema diagrams would help to crystallize this and other relationships, but during the course of the study some students still had not experienced the aha moment that would make this knowledge their own.

In step four: Solve, the students performed the math computation, and most of the students did this correctly. There were some times when calculators were used. Many times the numbers to be added or subtracted were low enough to be easy for the students. I did have to remind the students to subtract from the higher number during one lesson. One student, Pat, who lagged behind the others in computation skills, made most of the computation errors.

The procedures of the class and the materials used to facilitate dialogic problem solving can either help or hinder the process of solving word problems. At first, students were confused rather than supported by the strategy. The lengthy metacognitive questions on the checklists were initially overwhelming. Maybe there were too many words for inexpert readers. But, after they were familiar with the FOPS steps, the students began to follow the steps on the checklists, often without looking at the checklists.

Perhaps some students experienced "cognitive overload" with the strategy and the checklists. There were many new things to learn just to decide if adding or subtracting was indicated. The schema strategy clearly indicated student's missing vocabulary and concept knowledge. The diagrams were not readily understandable to them because of this missing understanding. The students struggled with the mapping because they did not understand the relationship of the *total*, or *big number*, to all the other numbers in the problem. The students also struggled with the strategy because they had trouble interpreting the semantic elements in the word problems. They might have been distracted from solving the word problems because their minds were so engaged in negotiating the maps and steps of the schema strategy.

Time was a factor in what the students learned. We had about two months time to learn and practice the new skills of dialogic problems solving and mapping in the schema diagrams during the research study. In this time, the students memorized the strategy steps and mapping. They learned to follow the steps and engage in dialogue about how to solve the problems. But they did not improve in solving rich or multi-step word problems. They recognized patterns and remembered some vocabulary concepts, but overall growth in accuracy in solving word problems from beginning to end was minimal. Table 5 and Table 6 on the following pages show the students' scores during the study and describe the content of the quizzes.

The most difficult quizzes from the middle of the study (Quizzes 7, 8, and 9) compared to the most difficult quizzes at the end of the study (Quizzes 16, 17, and 18) show similar achievement in the mean student scores on Table 6. A change from 76% to 78% is not significant growth. A positive note is that the independent work on Quizzes 17 and 18 was as good as the partner work on the other quizzes in this comparison.

When the individual scores for the same quizzes are compared on Table 5, two students' scores improved and two students' scores declined. Terry's scores went from 75% on quizzes 7, 8, and 9 to 84% on quizzes 16, 17, and 18. Ben's scores changed from 80% to 85%. Pat's scores started out at 73% and ended with 69%. Andy's scores started at 77% and ended with 75%. His low score of 67% on Quiz 17 was partly due to his need for more time. In order to finish the three problems during class, he skipped the mapping and labeling steps. He lost points and made the wrong decision at the planning step for one problem. Pat's emotional issues may have affected his achievements. His participation was sometimes on-task and sometimes silly.

Table 5. *Word Problem Quizzes: Schema Diagram Quizzes – Student Scores*

Quizzes varied in mode of working, content of the problems, and level of difficulty.

Quality Scoring: Each problem worth 3 points - 1 point given for each problem solving step:

Organize – map correctly, Plan – create correct number sentence, Solve – compute correctly

Quizzes: 12 points per quiz

Quizzes	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
Terry	100	88	83	92	71	78	70	73	83	100	ab	100	100	58	100	87	100	67
Pat	88	*25	83	82	58	78	70	73	75	83	100	100	33	83	87	63	83	60
Ben	100	92	92	82	71	99	80	67	92	100	100	100	100	33	90	87	83	86
Andy	75	75	92	82	71	78	80	67	83	83	100	100	50	78	93	75	67	83

*Note: Percentage correct, * student slept during quiz*

Quiz 18: *Follow-up, given 3 weeks after research ended, independent.*

Table 6. *Word Problem Quizzes: Schema Diagram Quizzes – Descriptive Information*

Quizzes varied in mode of working, content of the problems, and level of difficulty.

Quizzes: 12 points per quiz

Quizzes	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	***18***
Mode	P	I	P	I	P	P	P	P	P	I	P	I	I	I	P	P	I	I
Content	C	C	C	C	G	G	C	C	C	C	C	C	C	C	C	C	C	C
							G	G	G		G	G	G	G	G	G	G	G
																CM	CM	CM
Difficulty	1	1	1	1	1	1	2	2	2	1	2	1	2	3	2	3	2	3
Response	91	70	88	84	68	83	75	70	83	92	100	100	71	63	93	78	83	74

Quiz 18: *Follow-up, given 3 weeks after research ended*

Note: *Mode: P = Partner, I = Independent; Content: C = Change, G = Group, CM = Compare;*

Difficulty: 1 = one variable, 2 = two variables, 3 = three variables;

Response: Mean percentage of all students' scores.

The problems presented in the Math Center were considered "rich" problems because they could be solved in more than one way. They varied in difficulty, with white cards easiest, blue cards of average difficulty, and red cards most difficult. The students received a mix of cards in each Center experience, and their achievement was not related to the difficulty level determined by the cards' publishers. Sometimes they missed the easy problems and correctly solved the hard ones. Ben and Andy liked to solve the problems involving geometry. They succeeded at those tasks even when they were complex. Terry was often partially successful. She would quickly choose a workable method for solving a problem, but she would struggle to get the accurate answer. Pat's understanding was unpredictable and inconsistent. Table 7 on the following page portrays the students' scores on Math Center problems.

I constructed the Math Center table by recording the students' scores in the order in which they solved them. The groupings vary for each student because they worked with different partners and on different problems at each session. The general difficulty of the tasks remained constant. Some students completed more problems than others in the time they had. I noticed that the students' scores dipped lower in the middle of the study, then lifted in the end. I interpreted this to portray an increase in confusion in the middle of the study, and a hopeful upturn at the end, as some confusion was replaced with understanding.

Table 7. *Center "Rich" Problems – Scores*

Third grade level problem cards required more than one step or creative thinking to solve. Students worked in pairs. Levels of teacher help decreased, beginning with set 3.

Problem sets	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Terry	60	100	60	60	60	60	40	40	60	80	80	80
Pat	60	100	80	40	40	40	60	100	60	80	x	80
Ben	80	80	20	40	80	60	100	100	x	x	x	60
Andy	80	80	0	40	60	80	60	x	x	x	x	60

Note-Percentage correct; Problem Sets: 5 problems in each set.

Note: x *indicates student did not take quiz.*

Set 12*: Follow-up, given 3 weeks after research ended, independent.*

At the beginning and end of the study, the students responded to survey questions about their feelings toward solving word problems and about the schema diagram strategy. Table 8 shows how the students evaluated their own success in problem solving at the beginning of the study and at the end of the study.

Table 8. *Student Survey Results*

Questions:	*Begin* / *End*	*Poor* *Not at all*	*OK* *A little*	*Good* *Pretty much*	*Very good* *a lot*
Describe your math skills.	B			2	2
	E		1	3	
Describe your math grades.	B		1	2	1
	E		1	2	1
How well do you solve math word problems?	B	1	1		2
	E		1	2	1
Do you like to solve math word problems?	B		2	1	1
	E			1	3
Do you like math?	B		2		2
	E			1	3
Do you like easy word problems?	B			2	2
	E			1	3
Do you like hard word problems?	B	1	1	1	1
	E		2	2	
Do you have a plan to use when you solve word problems?	B		2	2	
	E		1	2	1
How well does your plan work to help you solve the problems?	B*		1	2	
	E		1	3	
I think it is important to be good at solving word problems.	B			2	2
	E				4
Total responses:	B*	2	10	12	13
	E	0	6	12	16

Positive Feedback Percentages:	*Beginning*	*Ending*
	73%	89%

Note: *One student did not respond

Table 8. Survey Results (cont.)

Strategies That Students Like to Use to Solve Word Problems		
Strategy	*Beginning*	*Ending*
Draw a picture	2	0
Circle facts in the problem	1	2
Use a calculator	0	4
Work with a partner	3	4
Explain my work	1	2
Guess and check	1	0
Write a math sentence	1	2
Use a list of steps	1	2
Add the numbers	3	1
Use a problem map/ diagram	0	2

Is it important to be good at solving word problems? Explain.

Pat: B - Yes, I circle a lot because it is important for the question.

E - Yes, because if you don't know how when you are little, you won't know when you are big.

Ben: B - Yes, because sometimes it is fun and sometimes we get to color.

E - Yes, you should know all of it.

Terry: B - Yes, I like it but not that much because they are hard.

E - No, because all people could do it.

Andy: B - Yes, or you won't learn math.

E - Yes, because it is nice. It is cool.

The students reported higher percentage of positive responses towards math word problem solving at the end of the study than at the beginning. No-one rated themselves poor in skills at the end, and they all liked easy problems. No-one liked hard problems at the end. They were split on the effectiveness of the plan. When I asked the students if the schema diagrams helped them, two students said, yes; one said, a little bit; and one said, not at all

The students were sometimes confused while they were solving word problems and when we were discussing word problems. Ben began to tell me when he was confused. One time, while I was explaining for another student who did not yet understand, he said, "I understand it, but when you keep explaining, it my head feels like it will explode." Ben's comment is a great example of self-advocacy. I praised him for speaking out for himself, and I stopped explaining to the group when one person needed clarification. For children with language learning disabilities (LLD), too much language is not helpful. Stone (2002) emphasizes that implementing scaffolded instruction for children with LLD is "fraught with difficulties" (p. 185). "One obvious issue for children with language difficulties is the overreliance on the verbal channel of discourse exchanges to carry the instructional "force" of scaffolding". (Stone, 2002, p. 185)

Stone (2002) calls for a "greater attention on the part of both researchers and practitioners to the dynamics of instructional activities targeted at atypical children" (p. 193).

> All the key ingredients must be in place. Although it is important to break down the task into sequential substeps, the adult must do more. In addition s/he must facilitate the rebuilding or constructing of the task. In the process the adult must empower the child (i.e., invite active investment and inference). Also, it is important to be mindful of potential impediments to the child's "uptake" of the adult's invitations to inference. Conceived in this way, effective instruction (and learning) rests on can interpersonal communication and thus is social at root. Because of its social nature, and because of the pragmatic inferences involved, emphasis must be placed on the fostering of shared perspectives (on the task at hand). Equally important is the issue of interpersonal trust."
>
> (Stone, 2002, p. 185)

I realized that I could use the schema representation diagrams to illustrate concepts, while using fewer words.

During my study, the diagrams and the strategy checklist anchored the discourse within a scaffold of support. The conversations in pairs and in the whole group formed a supportive discourse and started the students along a path towards mathematical communication and understanding.

As I considered the tools I had used to guide students to organize, plan, and solve all kinds of word problems, I formed an image of our journey from the metaphors embedded in my field log.

I saw an image of myself rushing along a path in the woods with sometimes eager, sometimes reluctant, children trailing after me. Words such as rushing, guiding, and struggling, described my actions and feelings. The students agreed and disagreed, were rigid and flexible, rushed and lagged behind, understood and were confused, rejoiced and shut down, gave up and persevered. I had the map in my hands, and I led them along the way. At the end of the study, we were still under the trees, but I imagined some sunlight streaming through the canopy. The students had their own maps, but they found that the map was hard to read. They could follow the guide and find their way with a partner, but on their own, their success was less sure.

ACTION RESEARCH - THE NEXT STEPS

This study was my first attempt to use these representational schema diagrams to teach word problem solving. I was learning along with the students, and I usually chose to adhere to the recommended sequence, because it represented a greater knowledge than my own. The diagrams assisted me perhaps more than they helped the students. I could understand the elegance of the diagrams in mapping the relationships in the word problems. I found the strategy helpful as a diagnostic tool. It helped me to identify the vocabulary and concepts that were not well understood by the students.

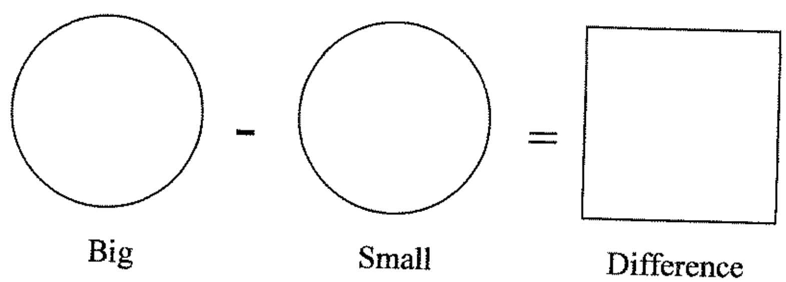
I need to make the diagrams more meaningful to students. I considered adapting the way I introduced the diagrams. The diagrams could be introduced as illustrations of key relationships between numbers. Later they could be used to show the same relationships in word problems. Since the schema diagrams illustrate key concepts or *big ideas* such as total, big group and small group, sum, and difference, the diagrams could be helpful as illustrations, and later as tools for creating number sentences for word problems. This shifts the focus from learning a new procedure to learning how to represent and solve word problems.

By linking the diagrams to vocabulary and concept instruction, I can emphasize the relationships in the problem types they are learning within the curriculum. Instead of arbitrarily starting with the Change diagram, I will plan my use of the diagrams in relation to the problems the students encounter within the math instruction. In this way, I can embed them in the curriculum rather than making word problem instruction a separate entity.

The students remembered certain things about the diagrams. They remembered that Group meant to add, and they connected Change with subtract. Later when I taught the Compare schema, they connected Compare to subtract. This tendency of the students to simplify in order to understand is a guide to effective instruction for them. This may signal that the complexity was not stimulating, but confusing for some students. If the simplified understanding is needed as a base, then I need to take small steps toward the complex understanding the schema represent.

After the study ended, I continued to practice problem solving with the students. My class size had nearly doubled, which meant that three of the seven students did not have a background in the schema diagram strategy. Teaching the Compare strategy gave me a chance to reteach the problem solving steps and to introduce the schema diagrams to everyone. I wanted to teach the students about the Compare diagram because I had seen in their Center work that they did not connect comparison questions with subtraction. The students understood the concept of comparing to determine greater than and less than, but they did not understand the connection between subtraction and comparison. I introduced the compare lesson by comparing my height to the height of one of the students. They could easily answer who was taller than or shorter than the other. I indicated the empty space between the top of the student's head and the top of my head, and told them that this distance tells how much taller I am. I asked the students for the word for that distance, and Pat volunteered the correct term – the difference. I led the students to figure out that the answer to a subtraction problem is called the difference, as the answer to an addition problem is called the sum. We discussed how we could find out the number for this difference. The labels on the Compare diagram are big, small, and difference.

Figure 14. *Compare Diagram*



Then we discussed the key words that indicate a Compare situation. Beginning with more than and less than, we generated lots of comparisons – bigger than, heavier than, older than, etc. I stated a rule for Compare problems – If you find comparison words like more than or later than, you subtract to find the difference. We found Compare language in word problems and highlighted the terms. Over the course of two weeks, we practiced with the schema diagram and with comparisons using charts and graphs. They worked in pairs or in the whole group.

This way of introducing the Compare schema is different from the way I taught the other schema diagrams during the study. In this model, the meaning of comparing two amounts is demonstrated with a real-life connection. Then, the schema diagram is added to portray the comparison relationship. During the study, I used meaningful stories to explain the diagrams. This way, I started with a meaning they knew, made a real-life connection, and focused on the extended meaning represented by the diagram. The emphasis was on the meaning of Compare situations rather than focusing on the Compare diagram tool.

I found that the students remembered the rule and the key words that indicated that rule. I heard them say to each other, "It is Compare, so we subtract." I heard them say, "Put the *big number* on top." I heard them label correctly in the answer: Tom has six more marbles than Sue.

Later, I modeled the trickier problems, when the difference was given, and addition was indicated to find out the *big number*. We also talked about problems where the difference was given, but we still subtracted because we needed the other small group, since the *big number* was given. These relationships were more complex than the original idea of: subtract when you compare.

The students approached these complex problems with varying degrees of success. When they encountered complexity, they could not rely on the rule that guided the original understanding. Sometimes they analyzed the problem accurately, mapped correctly, and used the schema to help them decide to add or subtract. They had the most success when the numbers used in the questions were lower than twenty. My interpretation of this was that they could visualize the smaller amounts to assist them in using the schema diagram. Essentially, this was the same difficulty they had had with the Change and Group diagrams.

I concluded that the diagrams were not simple tools for the students. The diagrams were meaningful to me, but less meaningful to my students. Two of the students said that the teacher should continue to use the diagrams because they helped her. They must have noticed my enthusiasm for the maps! I wondered what explained this difficulty and what I could do to bridge it.

To understand why the students had problems, I turned to literature on teaching and learning. Ellis and Worthington (1994) state this as a principle of effective teaching: "The critical forms of knowledge associated with strategic learning are: (a) declarative knowledge, (b) procedural knowledge, and (c) conditional knowledge. Each of these must be addressed if students are to become independent, self-regulated learners" (p. 35).

Declarative knowledge is factual information, such as knowing that the *difference* is the answer to a subtraction problem. This is also termed semantic knowledge. Procedural knowledge refers to the "degree of familiarity a student has for the steps of a given strategy" (Ellis & Worthington, 1994). Conditional knowledge is knowing when and where to use a strategy.

For students to succeed in using a strategy, they need to proceed from the simpler level to the complex level. In my study, the students' declarative knowledge was known, but not applied. They learned the procedural steps to use the strategy of mapping, and they followed the FOPS strategy checklist routine to discuss and map, but they were not successful at the conditional level. They inconsistently applied the skills and they made errors in choosing the problem type and in mapping.

Ellis and Worthington (1994) refer to the work of Bransford, Vye, Kinzer, and Risko (1990) who stress that educators must teach conditional knowledge as well as declarative and procedural knowledge. "When information is merely memorized, it will remain inert and fail to transfer to potentially relevant situations." (p. 41). Bransford et al. (1990) stressed that educators must guard against teaching content in a "rote, highly context specific way" (p. 41). Just as we can remember ten numbers in a sequence because it is a phone number, imbuing tasks with meaning enhances the student's ability to organize and recall the information. School tasks should be presented in meaningful contexts (Bransford et al., 1990, as cited in Ellis & Worthington, 1994).

As I looked toward future problem solving tasks for my students, I brainstormed ideas to make the tasks more meaningful. Westman (1990, as cited in Ellis & Worthington, 1994) discussed cognitive rigidity and flexibility. I had identified this as a frustration during my study. My efforts to get my students to "tune-in" to the complexity of relationships represented by the schema diagrams were met with resistance. While cognitively flexible children vary their strategies according to task demands, children with rigid cognitive styles "may be able to activate their knowledge only in very specific contexts, namely those which closely resemble the original learning situation" (Ellis & Worthington, 1994, p. 41). This matches my description of my students' performance as they worked with the schema diagram strategy. Worthington and Ellis write:

> Bareiter and Scardamalia (1985) have stressed that instruction designed to teach both knowledge acquisition and utilization will inevitably fail if direct efforts are not made to provide students (with opportunities) to use their acquired knowledge flexibly in solving a variety of real-life problems. (p. 41)

In real life, most problems require flexible use of knowledge, but many school tasks do not demand cognitive flexibility.

Cognitive rigidity can produce strategies that serve the students' purposes to "get by" (Bransford & Vye, 1989, as cited in Ellis & Worthington, 1994). Students with learning disabilities sometimes rely on self-taught coping strategies to get by. Two of the students in my study seemed to fit the category of rigid thinkers. In my field log codes and analytic memos, I had referred to this as a frustration, sometimes calling it concrete thinking.

As I plan to go forward, I want to continue to push my students towards flexible thinking. In order to jolt them into cognitive action, I want to incorporate real life applications into problem solving activities. By carrying out real tasks, perhaps they will go beyond the procedural routine. The tasks have to be within their ability with a partner, yet demanding enough to require organizing information and using a strategy to resolve the problem.

I tried out the idea of using a real task for the students to accomplish. Money skills are real-life skills, and real money can buy real things. The students know those facts. They also know how to count money, although it is not easy for them to keep large amounts organized. I devised a real-life word problem that they could solve. Each group would receive a bag of change, and they would count the money in their bag. Together they would determine the total amount of money available. The total amount would be used to plan a class luncheon for the math group. They could use menus from local restaurants that deliver their food. They would decide which restaurant to buy from and how much food to buy to feed the whole group. The successful execution of this process would result in a luncheon party. As the class worked each step of the problem, we could represent it in mathematical language – using the schema diagrams and number sentences.

One experience like this will not be enough, but many experiences like this may produce flexible problem solvers. For some learners, the problems have to be carefully scaffolded in many small steps toward complex thinking. The need for flexible thinking may not be apparent in abstract paper and pencil based word problem challenges. Planning real-life problems to be solved in small, planned steps, then connecting the planning process to paper and pencil schema representations through dialogue, may be an effective way to build complex and flexible thinking.

I plan to be flexible in the ways I introduce the diagrams. The sequence of introducing them can be determined by the sequence of the classroom math curriculum. By embedding the schema diagrams in the curriculum and connecting them to real-life demonstration of important mathematical relationships, I can enhance the meaning they have for the students.

Important math concepts can be explained with the diagrams. The schema diagrams can portray mathematical relationships such as: equivalence, comparison, sum, difference, and total. It is essential for students to clearly understand these big ideas in mathematical thinking. The schema diagrams could be valuable tools for illustrating some key understandings, such as the big/small comparison relationship and the relationship of small groups to a total. I can introduce the schema diagrams to my students as illustrations of mathematical relationships, instead of only focusing on creating number sentences from the mapping of facts in word problems. Word problems can be linked to the math concepts that are the key understandings needed to find a solution.

The pace of introducing complexity will have to be determined by the students' responses. For students who are easily confused, I need to determine the key relationships and concepts that they do not understand, such as the total or big number, and choose or devise word problems that illustrate the key concepts that they need to learn. After students work to represent the problem, class discussion should help to clarify the ideas the students discussed with their small group.

Word problems with more complex language can be introduced after the key relationships are understood. Teaching the students to use the diagrams to analyze new semantic patterns will promote flexible thinking. Allowing them to build a foundation of basic understandings before introducing variations may allow for greater success.

I don't mean to have less problem solving during the school week, but I mean to have the students solve fewer problems wrong. The rich word problems presented in the center activity during the study were explained piecemeal to the students as I worked with the pairs. This seems haphazard to me as I reflect on the variability of their scores. I think it will be more valuable to present one rich word problem from the math center to all the students, give them time to work with partners or on their own to solve it, then immediately discuss their approaches. It is important for the students' to have more consistent, corrective feedback than I provided during my research.

Carefully choosing rich word problems that relate to topics the students are learning in the math curriculum and planning for the conversation will enhance the development of mathematical discourse. Teacher questions cannot be wholly preplanned, since the questions should be in response to what students say (NCTM, 1991). Yet, anticipating what can be learned and what concepts the dialogue should clarify can help the teacher plan questions that lead students to make connections and elicit knowledge that is relevant to the problem (NCTM, 1991). The questions should guide students to figure out how the pieces of their knowledge fit together and lead to explanations and proofs of their solutions. Through mathematical discourse students should learn that an answer is right if it can be explained or proved (NCTM, 1991).

It is important to keep the goal of instruction clear. The goal needs to be increased mathematical understanding, improved skill in solving mathematical word problems, and the development of a mathematical disposition in each student. Learning to use a strategy is a legitimate sub-goal.

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APPENDIXES

Appendix A: Impact of Neurological Disorders on Learners

Section II **Academic Interventions**

Teachers and parents often have difficulty distinguishing how the different disorders affect academic functioning. The chart below provides examples of how the neurological disorders impact learning.

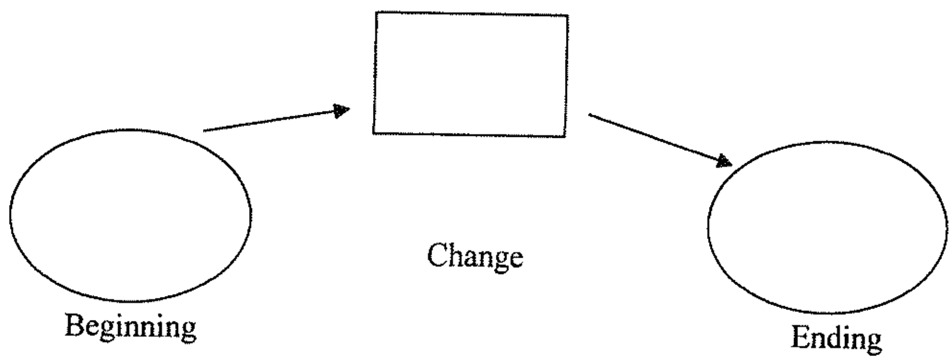
Impact of Neurological Disorders on Learners

	ADHD/ADD	TS	OCD	LD
Learning	Unable to apply learning.	TS symptoms disrupt learning process. Energy to suppress tics preempts learning.	Anxiety or OCD symptoms interfere with learning. Student "stuck."	Information unlearned due to memory impairment.
Memory	Information stored in cognitive file. Lacks organized strategies for storing, accessing information due to forgetfulness, impulsiveness and disorganization.	Pieces of information missing in cognitive file due to TS symptoms interfering with learning by disrupting ability to store information in memory.	Pieces of information missing due to OCD symptoms interfering with learning by disrupting storage process.	Information not stored in cognitive file, or filed incorrectly and cannot be retrieved. Does not know how to label or store information.
Strategies	Does not attend to learn strategies or does not consistently generate strategies.	Understands strategies, able to use strategies except when TS symptoms interfere with ability to use strategies.	Understands strategies except when OCD symptoms interfere with ability to use strategies.	Does not always understand or generate strategies.
Sequencing	Unable to attend or too impulsive to remember to use sequencing.	Able to sequence except when TS symptoms cause impulsivity which interferes with the ability to sequence.	Able to sequence except when OCD symptoms cause cognitive looping.	Difficulty sequencing.
Problem Solving	Unable to remain on-task long enough to problem solve.	Able to problem solve except when TS symptoms interfere with the ability to problem solve.	Able to problem solve except when OCD symptoms interfere with ability to problem solve.	Difficulty problem solving.
Social	Unaware when to use social cognition or when to trust own intuitions.	Social interactions affected by ignorance of others, embarrassment from TS symptoms, teasing, withdrawal.	Social interactions affected by anxiety and OCD symptoms.	Social cognition deficits.

(Dornbush & Pruitt, 1995)

Appendix B: Change Schema Diagram and Checklist (Jitendra & Hoff, 1996)

Change Schema Diagram



CHANGE PROBLEM CHECKLIST

Step 1. Find the problem type.

- ☐ Did I read and retell the problem?
- ☐ Did I ask if it is a change problem? (Did I look for the beginning, change, and ending? Do they all describe the same thing?)

Step 2. Organize the information using the change diagram.

- ☐ Did I underline the label that describes the beginning, change, and ending and write in label in the diagram?
- ☐ Did I underline important information, circle numbers, and write in numbers in the diagram?
- ☐ Did I write a "?" for what must be solved? (Did I find the question sentence?)

Step 3. Plan to solve the problem.

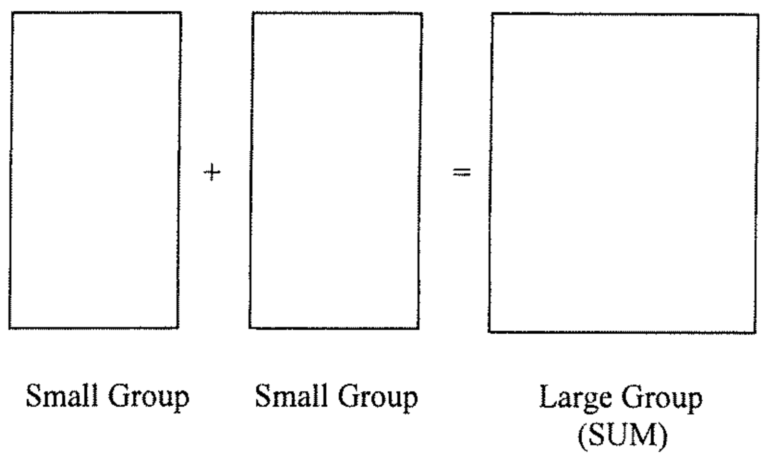
- ☐ Do I add or subtract? (If the "BIG" number is given, subtract. If the "BIG" number is not given, add)
- ☐ Did I write the math sentence?

Step 4. Solve the problem.

- ☐ Did I solve the math sentence?
- ☐ Did I write the complete answer?
- ☐ Did I check if the answer makes sense?

Appendix C: Group Schema Diagram and Checklist (Jitendra & Hoff, 1996)

Group Schema Diagram



GROUP PROBLEM CHECKLIST

Step 1. Find the problem type.

- [ ] Did I read and retell the problem?
- [ ] Did I ask if it is a group problem? (Did I look to see if two or more small groups combine to make up a large group?)

Step 2. Organize the information using the group diagram.

- [ ] Did I underline the large group and small groups and write in group names in the diagram?
- [ ] Did I circle numbers for the groups and write in numbers for groups in the diagram?
- [ ] Did I write a "?" for what must be solved? (Did I find the question sentence?)

Step 3. Plan to solve the problem.

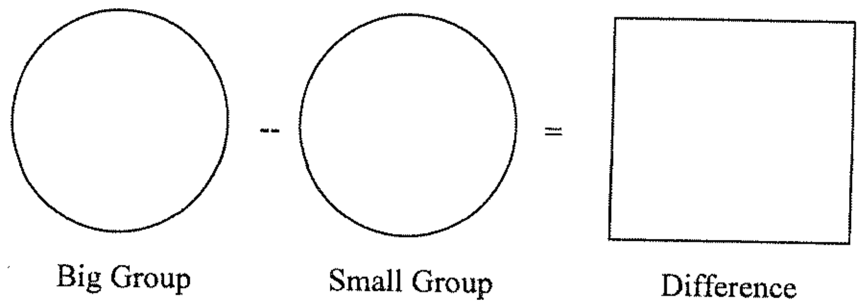
- [ ] Do I add or subtract? (If the "BIG" number is given, subtract. If the "BIG" number is not given, add)
- [ ] Did I write the math sentence?

Step 4. Solve the problem.

- [ ] Did I solve the math sentence?
- [ ] Did I write the complete answer?
- [ ] Did I check if the answer makes sense?

Appendix D: Compare Schema Diagram and Checklist (Jitendra & Hoff, 1996)

The Compare Schema



COMPARE PROBLEM CHECKLIST

Step 1. Find the problem type.

- [ ] Did I read and retell the problem?
- [ ] Did I ask if it is a compare problem? (Did I look for compare words – taller than, shorter than, more than, less than?)

Step 2. Organize the information using the compare diagram.

- [ ] Did I underline the comparison sentence or question and circle the two things compared?
- [ ] Did I reread the comparison sentence or question to ask, "Which is the "BIG" amount and the "SMALL" amount?" and write in names of things compared in the diagram?
- [ ] Did I underline important information, circle numbers and labels and write in numbers and labels in the diagram?
- [ ] Did I write a "?" for what must be solved? (Did I find the question sentence?)

Step 3. Plan to solve the problem.

- [ ] Do I add or subtract? (If the "BIG" number is given, subtract. If the "BIG" number is not given, add)
- [ ] Did I write the math sentence?

Step 4. Solve the problem.

- [ ] Did I solve the math sentence?
- [ ] Did I write the complete answer?
- [ ] Did I check if the answer makes sense?

Appendix E: Progress Monitoring Benchmarks

Oral Reading Fluency Norms (AIMSweb)

End of:	25th Percentile	50th Percentile
Grade 1	33 wcpm	52 wcpm
Grade 2	70 wcpm	94 wcpm
Grade 3	85 wcpm	110 wcpm
Grade 4	99 wcpm	124 wcpm
Grade 5	112 wcpm	137 wcpm
Grade 6	117 wcpm	145 wcpm
Grade 7	146 wcpm	167 wcpm
Grade 8	145 wcpm	171 wcpm

(www.aimsweb.com)

Math Computation Norms

Winter of:	25th Percentile	50th Percentile
Grade 1	4 dcpm	7 dcpm
Grade 2	6-8 dcpm	10-11 dcpm
Grade 3	6-8 dcpm	10-13 dcpm
Grade 4	6-8 dcpm	13-21 dcpm
Grade 5	6-8 dcpm	13-26 dcpm

(Shapiro, 2003)

Time Limits for Administering Math Computation Probes

Grade	Time (minutes)
Grade 1	2
Grade 2	2
Grade 3	3
Grade 4	3
Grade 5	5
Grade 6	6

(Fuchs, Hamlett, Fuchs, 1999)

Writing Fluency Norms Words Written Per 3 Minutes

Writing

Grade	Ranks	Fall	Winter	Spring
1	75	7	17	20
	50	4	13	16
	25	3	8	12
2	75	27	33	38
	50	21	25	28
	25	14	18	22
3	75	41	48	50
	50	33	41	42
	25	27	34	33
4	75	53	57	60
	50	45	48	57
	25	36	40	39
5	75	60	65	69
	50	51	55	57
	25	43	45	45

(Shapiro, 2001)

Appendix F: HSIRB Approval

1742

MORAVIAN COLLEGE

September 17, 2004

Margaret M. Scheihing
504 Arch Street
Pen Argyl, PA 18072
megscheih@hotmail.com

Dear Margaret Scheihing,

The Moravian College Human Subjects Internal Review Board approved your proposal: Improving elementary students' mathematical word-problem solving skills through dialogic strategy instruction. Given the materials submitted, your proposal received an expedited review. A copy of your proposal will remain with the HSIRB Chair.

/ Please note the phones numbers you have on both Informed Consents for Dr. Shosh may be incorrect.

Should any other aspect of your research change or extend past one year of the date of this letter, you must file those changes or extensions with the HSIRB before implementation.

This letter will be e-mailed and snail-mailed to you. Best of luck with your research.

James Barnes
Chair, Human Subjects Internal Review Board
Moravian College
610-861-1672 (voice)
610-861-1657 (FAX)
barnesj@moravian.edu

Appendix G: Principal's Approval

August 26, 2005

To Whom It May Concern:

I give my consent to Meg Scheihing to conduct a research study in her classroom as part of her course work to earn a master's degree in education from Moravian College. I understand that the methods to be used in the study are supported by current educational research literature about effective instructional strategies for learning disabled students. The focus of this study is a method for teaching students how to solve mathematical word problems.

To participate in this study, each child's parent or guardian must give informed consent by signing and returning the consent letter. For the sake of privacy when reporting the study results in the master's thesis, pseudonyms will be assigned to each participant student. Information referring to children's learning disabilities will be reported as group data. No student names will be used in the report.

All of the students in Mrs. Scheihing's math classes will be involved in the instruction about mathematical problem solving. All will learn how to use the diagrams to organize the numbers in word problems. Once the mathematical number sentences are written, students will use their arithmetic skills to solve the problems. Students will receive the support they need to succeed in the skills taught in the study. Everyone will complete 4 word problems independently at the end of each math session as a quiz. But, only the study participants opinions, experiences, and quiz grades will be reported in the thesis, though their names will not be used.

Finally, I am aware that questions regarding this research should be directed to Mrs. Scheihing at (484) 373-6220 or to her advisor, Dr. Joseph Shosh, at (610) 861-1842 or by e-mail at jshosh@moravian.edu.

Sincerely,

____

Principal, [redacted] Elementary School

Appendix H: Parent Permission

September 1, 2004

Dear Parents:

During the 2004 -2005 school year, I will be completing courses toward a Master's Degree in Curriculum and Instruction at Moravian College. Over the past three years, these courses have kept me up-to-date with the most effective practices of teaching.

Moravian's program requires that I conduct a careful study of my own teaching practices. The focus of my research this semester (September 1 to December 20) is a method for teaching students how to solve word problems. The students will learn to use diagrams to map out the numbers in a word problem and decide whether to add or subtract. They will use a checklist to proceed through four steps that guide their thinking and they will work in groups to discuss and explain their solutions.

The students in my math class will work on this problem solving for part of their math class three days per week. To check if they are making progress, I will sit with each child individually at the beginning of the study, in the middle, and at the end of the study to interview them and observe their strategies for solving problems. I will share the information I learn about your child's attitude toward problem solving, skill level, and progress in solving word problems on his or her quarterly progress report, or at any time you request.

All of the students in my fourth grade math class will be involved in the instructional and assessment activities as part of the class. However, participation in the research study is purely voluntary. Students may withdraw at any time and it will not affect their grade in any way. If they do withdraw, no information pertaining to your child will be including in the research report.

If students are involved, their opinions, achievements, and experiences will be included in the research report, but their names will not be used. All children's names will be kept strictly confidential. Pseudonyms will be assigned for the report, but students will not know their false name. General information about student disabilities will be reported as group data to protect your child's privacy.

My faculty sponsor is Dr. Joseph Shosh. He can be contacted at Moravian College by phone at (610) 861-1842 or by e-mail at jshosh@moravian.edu.

If you have any questions or concerns about my in-class research study, please feel free to contact me at school at (484)373-6220. Please sign and return the bottom portion of this letter to give your child permission to participate in this study. Thank-you for your help.

Sincerely,

Margaret M. Scheihing

I give my child______________________________ permission to participate in this research study about solving math word problems.

Parent or Legal Representative signature:__________________________ Date: __________

Appendix I: Word Problem Solving Steps (FOPS)

WORD PROBLEM SOLVING STEPS (FOPS)

Step 1. Find the problem type.

Step 2. Organize the information in the problem using the diagram (change, group, or compare).

Step 3. Plan to solve the problem.

Step 4. Solve the problem.

Appendix J: Student Survey

Student Survey

Directions: Read the statements and circle the answer that best describes you.

1. Describe your math skills.
   Poor OK Good Very Good

2. Describe your math grades:
   Poor OK Good Very Good

3. How well do you solve math word problems?
   Poor OK Good Very good

4. Do you like to solve Math word problems?
   Not at all a little pretty much a lot

5. Do you like math?
   Not at all a little pretty much a lot

6. Do you like easy word problems?
   Not at all a little pretty much a lot

7. Do you like hard word problems?
   Not at all a little pretty much a lot

8. Do you have a plan to use when you solve word problems?
   Not at all a little pretty much a lot

9. How well does your plan work to help you solve the problems?
   Poor OK Good Very Good

10. Circle the strategies that you like to use to solve word problems:

draw a picture	guess and check
circle facts in the problem	write a math sentence
use a calculator	use a list of steps
work with a partner	add the numbers
explain my work	use a problem map/diagram

11. I think it is important to be good at solving word problems:

    Not at all   a little   pretty much   a lot

12. Give reasons for your answer to #11: ______________________________

____

Appendix K: Mathematical Word Problem Solving Interview

Mathematical Word Problem Solving Assessment

Interview

Name________________________________ Age_____ Grade_____ Gender_____
Date______________________________ Placement________ Interview # ______
Strategy Levels Mastered____

Part I - Here are three math word problems and some tools you can use later to solve them. First, we will talk about what you already know about word problems. I have some questions to ask you about math word problems. I will write your answers down for you. Ready?

(K) 1. Look over at those math tools. What tools do you see there that you have used before ?

(A) 2. Do you like to use any of those math tools to solve math word problems? Which ones? Explain why or why not.

(A) 3. Do you like to solve word problems? Not at all a little pretty much a lot

(A) 4. Why or why not?

(SP) 5. Describe your math skills. Poor OK Good Very Good

(SP) 6. Describe your math grades. Poor OK Good Very Good

(SP) 7. Describe how well you solve math word problems.
Poor OK Good Very Good

(A) 8. Do you like Math? Not at all a little pretty much a lot

(A) 9. Why or why not?

(ST)10. What do you do to solve math word problems like the examples I showed you?

(ST)11. A strategy is a plan that people use to solve problems. Tell me about any strategies you use to solve math word problems.

PART II - Solving the word problems.

Introduction: Now you get a chance to solve three math word problems. If you have trouble reading or understanding the words, just ask me. I will give you the problems one at a time. When you finish one, I will give you the next one. You may talk with me about the problems as you figure them out. You must write your answer down, and it is best to show the work so we can remember how you solved it.

Word Problem #1 - Gail had 43 coins in her collection. Then she bought 11 more. How many coins does Gail have now?

Word Problem #2 - 65 students at Wilson Elementary School made projects for the science fair. 2nd graders made 22 projects and 3rd graders made 27 projects. All the rest were made by 4th grade students. How many science fair projects did the 4th grades make?

Word Problem #3 - My dad is 39 years old . He is 10 years older than my mom. How old is my mom?

Questions About Word Problems:

(K) 1. How do you read math word problems?

(R) 2. How many times do you read math word problems?

(R) 3. As you read, how do you help yourself understand the problem?

(ST)4. How did you decide what to do to solve this word problem?

(ST) 5. Why did you set it up this way? Explain what you did.

(ST) 6. What did you think in your head when you were doing this part of the problem?

(ST) 7. What labels did you use and why?

(ST) 8. How did you (*or* could you) check your answer to see if it was correct?

(Observer/ tester notes are written on the interview question sheet.)
(Students write on the word problem question sheet- one problem per sheet.)

Key to Codes:
K = Knowledge
A = Attitude
R = Reading

ST = Strategy Use
SP = Self perception