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ASSISTING STUDENTS DEVELOP THEIR MATHEMATICAL THINKING PROCESS THROUGH WRITING

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ABSTRACT

This qualitative action research study documents the observed and reported experiences of nineteen third grade students and their teacher as they explored how to facilitate mathematical thinking process through written expression. The author shares the NCTM's (2000) philosophy that indicates when students gain insight into their thinking they are more effectively able to connect different mathematical strategies to a variety of situations. All the participants received instruction on specific math communication strategies throughout the study within their regular education classroom in a suburban elementary school. The study examines how the students utilized the modeled strategies in explaining their mathematical thinking process as they progressed through the different math communication levels. The study focuses on the scaffolding methods that were implemented in instructing each of the students at his/her developmental level. The data suggests that the teacher needs to scaffold instruction not only in reference to the student's ability within the specific communication level but also in connection with his/her understanding of the specific math concept that he/she is being asked to explain. In addition, the study advocates the need for students to have the opportunity to naturally progress through the distinct communication stages in order to be more successful in independently explicating complex math topics at a later phase. Finally, the researcher ponders what additional strategies she can implement in assisting her students develop their mathematical thinking

process. She also questions what supplementary scaffolding methods she would need to employ in working with students who require specialized instruction and/or ESL students in communicating their mathematical thinking process.

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RESEARCHER'S STANCE

My personal connection with math dates back to my elementary years, and for as long as I can remember, I have always looked forward to math. Early in my elementary education when I moved to the United States from Portugal, it was the first subject area in which I felt success. The numerical operations of mathematics were not affected by the language barrier, and the key words that I was learning assisted me in being successful in problem solving, as well. The successful connections that I built early in my educational career continue to impact my view of math and how I can reach students through this medium.

Math was always the subject that I enjoyed the most. Not only did I enjoy the challenge of solving mathematical problems, but I also looked forward to working with my peers in arriving at solutions through discussing what steps we needed to take along the way. The passion that I developed in learning about mathematics continued to evolve into my own teaching practice. As I model and allow my students to explore different mathematical strategies, I look forward to seeing the students work together to arrive at their own solutions. I feel that mathematics is a subject that some students tend to shy away from, conceivably because some may not have been exposed to a variety of teaching methods that address the manner in which they learn the best. Even math communication, the topic of my research, can be addressed differently through the use of diagrams and examples, as well as the actual written component. Because of my own

experience, I have also been able to use mathematics as a way to connect with ESL students and assist them in using math as a springboard to feel successful in their educational process once again.

As teachers we tend to have areas for which we have a great passion, and I am no different. Hence, when the time came to select my research topic, I knew I wanted to focus on mathematics, the subject area which I have always had an interest in, both as a learner and educator. In addition, I wanted to focus on an area in which I could improve my teaching in order to assist my students to become better overall learners. I sought to study specific strategies that students would be able to adapt to a variety of learning situations. As I analyzed my first three years and carefully considered the prevailing factors that emerged, I determined I wanted to focus my study on incorporating writing within math instruction to improve my students' mathematical communication skills.

From my personal experience in the classroom, I have often found that students tend to steer clear of math questions in which they have to explain their thinking process through writing. With mathematical communication being an important component of the PSSA, I feel that the ability to use writing to explain one's mathematical processes can be very beneficial to students in becoming proficient problem solvers across a variety of situations. I have also observed that when students acquire the needed skills to write mathematically, they transfer the skills to future tasks. Student's lack of confidence in answering this type of

question serves as a stumbling block. I believe the lack of confidence is due to the fact they do not possess the strategies to complete the task.

As I have prepared students to take the third grade math PSSA test during my first two years of teaching third grade, I have witnessed, that as students practice their writing in math, their communication skills have improved. The students are better able to transfer their mathematical thinking and then be more successful in solving word problems. My pondering question, however, was how could I increase the students' confidence in mathematical problem solving and provide them with the additional writing techniques earlier in the school year.

As I reflected on my instruction of mathematical communication in the past, one of the reasons that I felt it took students longer to reach the final goal was that I did not present enough opportunities for students to use writing in mathematics in a non-threatening mathematical manner. Even though I often talked to students about their feelings and attitudes towards math, I did not provide a manner to address their apprehensions prior to writing about their mathematical thinking process. In addition, throughout my educational courses as well as my practice, I know that instruction at the elementary level often follows a sequential path: modeling, shared demonstration, guided practice, and finally independent practice (Routman, 2003). As I worked with my students in writing about their mathematical thinking process, I knew I needed to first model the strategies that they will then be able to use in their communication of

mathematical processes. Next, I would incorporate shared demonstration where the students and I work together in using a strategy with a particular problem. The latter two steps in instruction, guided practice and independent practice, develop at each student's individual rate with the different strategies. I acknowledged that students would vary on how long it would take them to employ a strategy on an independent level, and I would need to adapt my instruction in providing guided practice through a variety of settings from one on one to small group, as well as pair tutoring. When the students are able to utilize the strategies at an independent level, they will be able to problem solve as well as communicate mathematically as they explore different problem solving scenarios.

An additional consideration that I employed in conducting my study is that each student would approach communicating mathematically with his/her own individual style. As an educator I know that each student has his or her own strengths and weaknesses and these factors would directly determine the manner in which they learn. In my communicating mathematically instruction I would need to consider that each student would "write" in his or her own manner in accordance with unique and individual processes. Students may use symbols, examples, pictures, diagrams, and/or writing in their mathematical communication. While the students' final goal is to communicate their mathematical thinking process, they can each arrive at this explanation through different methods. That is why I believed presenting different strategies would be

essential, so that students could actively employ the different strategies in problem solving process.

Through my experience as a student and as a classroom teacher, I have come to the realization that self-confidence with the subject matter is one of the key factors in determining how successful the student will be. While teachers must have high expectations for their students' final products; they must be cautious not to set the students up for failure by presenting a topic in a manner in which the students may at first feel unsuccessful. As I conducted my research, I paid careful attention to my students' self-confidence in order to assist them in becoming more successful in communicating their mathematical reasoning. I embarked on my action research study hoping to become a better educator by discovering ways that I could assist my students in being more successful in communicating their mathematical thinking as active learners.

LITERATURE REVIEW

The National Council of Teachers of Mathematics (NCTM) reminds teachers that we are living in an environment in which mathematics plays a vital role in multiple facets of our daily routines (NCTM, 2000). As a result of mathematics continuing to become more prevalent in everyone's daily lives, NCTM reports, "those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures" (p.5). An essential component for students to be successful with mathematics is being able to organize their mathematical thinking effectively and being able to communicate their mathematical thinking with others. When students are able to effectively communicate their mathematical process they gain insights into their thinking and are better able to connect different mathematical strategies to a variety of situations (NCTM, 2000). Researchers as well as the NCTM support that incorporating writing within math instruction will assist students in developing their communication skills; in fact in *Principles and Standards for School Mathematics* NCTM states, "Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics" (p.128).

The notion that writing can assist students in constructing knowledge is rooted within educational theorists' philosophies. Thayer and Giebelhaus (2001) acknowledge Piaget, Vygotsky, and Emig, all of whom stress the importance of

using writing as a learning tool in constructing knowledge. Thayer and Giebelhaus point out that writing should not simply be a form of copying teacher ideas. In order for writing to be effective it should provide the students with an opportunity to communicate information using their own words as well as form associations between prior knowledge and newly learned information in order to construct new knowledge. Phillips and Crespo (1995) agree with Thayer and Giebelhaus by affirming that "written communication is of special importance because it provides students with a record of their own thinking and developing ideas" (p. 4).

Incorporating writing into math instruction can be done through a variety of processes. One way to manage writing during math instruction is through the use of learning logs. McIntosh and Draper (2001) indicate that the purpose of learning logs is for the students to reflect on the skills and strategies that they are learning. Students can use learning logs to explain a problem solving process, to write about new vocabulary and how it relates to items that they are learning, to justify an answer, or even to reflect on one's process in understanding particular skills and strategies. McIntosh and Draper indicate that students should write in their learning logs frequently throughout each week.

Another way to incorporate writing into math lessons is through the use of math journals. Burns and Silbey (2001) indicate a variety of ways in which journals can be utilized in the classroom. One way that teachers can adapt journals

into their classrooms is by having the students reflect daily on their learning progress for that specific day. An additional possible use of journals is to serve as a mechanism to save notes and daily problems. A third way they suggest to use journals is for students to write their thinking process for a specific problem. Burns and Silbey (2001) indicate that math journals help students to “stretch their thinking” (p. 18) into the complexities of math problems. Journals provide the students with a means to show their thinking that is occurring in their mind. By writing in journals the students are then able to evaluate the strategies that they were using in their problem solving processes (Burns & Silbey, 2001).

Burns (1995), Burns and Silbey (2001), Goldsby and Cozza (2002), Aspinwall and Aspinwall (2003), along with Shield and Swinson (1997) have all researched specific strategies for teachers to implement as they incorporate writing into math instruction to foster higher order mathematical thinking skills. Burns (1995) provides strategies to assist teachers to foster mathematical thinking through writing. First, she indicates it is important for the students to know the purpose of their writing. Teachers need to point out to students that by writing they will be able to enhance their math skills and also assist the teacher in knowing their thinking process. She suggests that teachers tell the students that the teachers are the audience of the writing and the students should always show what they are learning and what they understand in their writing. Burns indicates that student entries can be used during class instruction. Burns indicates that the

students, especially those that struggle with math journals, should discuss the topic before writing. Another means to assist struggling student is to provide prompts. Burns also suggests that teachers provide individual assistance to those students who don't know what to write. An additional suggestion is for teachers to post math word lists in their classroom so that the students can reference them as they write. A final suggestion that Burns indicates is to have students occasionally revise and edit their entries. Overall Burns(1995) indicates that writing in math assists teachers in evaluating their teaching and assessing student progress.

Goldsby and Cozza (2002) suggest an additional strategy to incorporate writing as a venue to assist students in thinking mathematically. They illustrate how writing samples can be used as a "vehicle for mathematical learning" (p. 520), sharing ways through which teachers can use student responses to review previous instruction. In addition teachers can also use students' samples to demonstrate how students used different approaches to reach the same answer to show that each student's method is valid (Goldsby & Cozza, 2002). The NCTM concurs in this perception by stating, "By listening carefully to others, students can become aware of alternative perspectives and strategies" (p.128).

Aspinwall and Aspinwall (2003) researched how open-ended prompts can be used to assist students in developing their mathematical thinking. They found that open-ended prompts "allow teachers to explore the nature of students' understanding and to use this information in planning instruction" (p. 352). They

indicate that teachers can use the students' writing to decipher each individual learning process. Evans (1984) concurs with Aspinwall and Aspinwall's notion that writing can be used to assess a student's competency level with specific math concepts. In her study she investigated if elementary students could potentially use writing as a tool to learn mathematics. Evans indicates that writing is a more accurate assessment to gauge the student's understanding of mathematical concepts than tests.

Shield and Swinson (1997) describe a few writing activities to "encourage learning in mathematics through writing" (p.4). First, they share how writing a letter to a friend summarizing a topic that the student has recently learned can help him/her determine the needed information to assist his/her friend understand the concept. Second, Shield and Swinson indicate how explaining examples, both personal as well as samples within textbooks, can assist in "developing students' abilities to explain and justify the individual steps in a mathematical procedure" (pg. 5). Another strategy that they share is for students to compose their own definition for a particular math term. They illustrate how students need to classify the information they already know and identify what information is essential about the object in order to compose a definition that is understandable to their audience. Finally, Shield and Swinson share a graphic organizer called a link-sheet where students make connections to a mathematical concept. The link-sheet has four sections: math example; everyday sample; diagram, picture, graph; and

my explanation. They indicate that students will need teacher modeling before they can successfully complete all components; however, through practice and time, Shield and Swinson feel the students will be able to provide a comprehensive understanding of their knowledge through their connections using the link-sheet.

Regardless of the strategies or venues through which writing is incorporated into math instruction, researchers concur that teacher feedback is key to its success. Burns and Silbey (2001) indicate that students can orally share their entries with one another and provide constructive feedback to one another or teachers can write feedback on specific journal entries. McIntosh and Draper (2001) state that teachers should provide the students with positive responses in their logs in which they validate the students' thinking process. In addition, they suggest that teachers can give students topics to keep in mind for future responses. McIntosh and Draper also indicate that while the logs should not be graded numerically, effort grades can be used to show that teachers will not accept partial entries. Phillip and Crespo (1995) attribute the impact of feedback to the findings of their study in which they paired college students with fourth graders in a pen pal letter exchange during a three-month period. Phillips and Crespo determined that pen pal letters provide a means through which students can be more successful in increasing their quality of written mathematical communication.

They stated that pen pal letters established a setting through which students could dialogue and receive feedback on their thoughts (Phillips & Crespo, 1995).

Many researchers support the notion that using writing to assist students to develop their mathematical communication skills will help the students enhance their problem solving skills as well as increase math assessment scores. In his research Williams (2003) showed that students who are using a writing component in their math classes scored higher in problem solving performance tasks than students who did not have a writing component in their math instruction. He describes the need for students to use writing in order to concretely investigate their program solving process. In his study the treatment group contained twenty-two subjects, and the control group consisted of twenty students. All of the students were commencing their algebra study. In his research, Williams used the pretest and posttest method for comparing results. Both groups also received approximately five additional homework problems per week in which they needed to use higher thinking skills and math strategies to solve them successfully. Each response was then graded on a scale of four points each with descriptive requirements. While the students in the treatment group were required to write about their process as they solved the weekly problems, the students in the control group were not required to write their thinking for any of their steps in solving the weekly problems. Williams (2003) discovered that the students who were in the treatment group in which they were exposed to writing

in their mathematical instruction consistently scored higher by writing their solving progress for each response.

Thayer's (2001) experiment demonstrates how incorporating writing into math instruction can improve students' math scores. Thayer's quasi-experimental study took place in a suburban Midwest high school during a four-week period. The study focused on two heterogeneous geometry classes. One class had 21 students while the other had 22 students. Both classes consisted of students ranging from ninth to twelfth grade and were consistent in gender as well as student ability grouping. Both classes met for ninety minutes per day, and the same teacher taught the same geometry content to both classes. The only instruction that differed between the two classes was the use of writing. While in the experimental classroom the teacher provided the students with three to five minutes at the conclusion of class to write in their journals, the control group used the three to five minute time period to complete practice problems. Each student in both classes took the same pre and posttest. The experimental group had a slightly lower mean score on both pretests. However, the experimental group also had a higher mean score on both posttests. Through her analyses of the pre and posttest, Thayer also discovered that the writing was particularly helpful for the students who had the lowest pretest scores. Thayer concludes her study by acknowledging the fact that as more and more states continue to require state assessments and schools look for ways to improve math scores, writing is an

important learning tool that can be implemented to assist students in constructing knowledge.

Baxter, Woodward, and Olson (2005) discovered that writing goes beyond assisting students in understanding their mathematical thinking skills. They indicate that writing can be particularly beneficial to students who lack self-confidence towards sharing their thinking through oral discussions in the classroom, especially struggling students. The researchers also stress the importance of the affective response that can be created between students and teachers through written communication. The personal contact that the writing provided the students with the teacher helped the students to gain self-confidence and provided a positive dialog between the student and the teacher. Koirala (2002) also found that using writing provides a safe communication method for shy students who often do not volunteer during oral discussions.

Furner and Duffy (2002) studied an additional positive outcome that arises from writing during mathematics instruction. They indicate that there is a correlation between low achievement in math and math anxiety. Therefore, they feel that math instruction needs to change in order to provide an environment in which the students feel safe to explore with a variety of mathematical teaching strategies. One way to provide students with a secure atmosphere to explore mathematics is by incorporating writing into math instruction. They indicate writing can provide students with the opportunity to communicate their honest

thoughts about math and how and when these thoughts originated. When students are able to address their dispositions towards math, teachers can understand their students' feelings and apprehensions prior to introducing new topics, and therefore can plan their lessons accordingly. By reading students' entries, teachers can then assess if the students are having difficulties with particular topics and address the students' apprehensions. Furner and Duffy feel that it is imperative for teachers to acknowledge students' anxiety toward math and address the source of the apprehension in order to assist students in having a positive outlook towards math. In establishing a safe learning environment they feel teachers need to have a variety of teaching strategies as well as a diversity of assessment methods as part of math instruction.

One of the disadvantages educators are concerned with incorporating writing into math instruction is the lack of time (Burns & Silbey, 2001). Teachers report feeling stressed by making time for the writing component as well as time to provide feedback for the students. One way that Ryan and Rillero (1996) suggest to assist teachers with the time constraint is to use the writing as an opening activity. They demonstrate that the writing will help student get right on task from the commencement of math class. Ryan and Rillero (1996) recommend that teachers select a prompt to review a previous day's topic, serving as a refresher by beginning new lesson. They also recommend that the prompt assist students in recognizing connections to everyday situations in which math are

used. The NCTM's recommendations are in agreement with Ryan and Rillero's (1997) suggestion that students need to be able to relate math topic to everyday use. While teachers often agree that writing is a valuable tool for students, especially in showing their understanding of particular topics, there are some disadvantages that discourage teachers from using writing, especially lack of time. Many researchers address the time issue by providing suggestions for teachers to use in creating a schedule that would be appropriate in their classrooms.

As the NCTM (2000) indicates, developing the student's ability to communicate mathematically is a crucial ingredient in assisting students to be more successful in understanding mathematic concepts. Current research suggests that using a form of writing in their math instruction may assist students in developing their mathematical thinking skills. Burns (1995) states that writing "is a window into what they understand" (p. 40). Burns and Silbey (2001) continue their analyzes by explaining that "when children write . . . they examine, express, and keep track of their reasoning, which is especially useful when ideas are too complex to keep in their heads" (p. 18). Dusterhooff (1995) concurs with Burns. She perceives writing as a way to help the students connect concepts and learn how different topics are interconnected. Manning and Manning (1996), too, discovered the positive outcome writing has in developing mathematical thinking skills. They stressed that "writing is a powerful tool for thinking and learning in math" (p. 107). In addition, Goldsby and Cozza (2002) indicate that writing can

be used to assess student learning in a more authentic way because it gives teachers a clear view into what the students are thinking.

TRUSTWORTHINESS STATEMENT

According to Arhar, Holly, and Kasten (2001), “Building a classroom environment based on respect is the first ingredient of ensuring confidentiality” (p.171). In order to create an environment of respect, I presented and described my study to the students in a class meeting, a comfortable setting where students are encouraged to share their opinions and beliefs. Part of the discussion focused on my role as a student. I shared with the students that teachers continue to learn new strategies in order discover the best way to assist students in being successful. I informed the students that they would have a pseudonym and their real names would not be used in the final report of the data. This not only provided anonymity in my study but also helped alleviate any apprehensions the students might have felt about having their work shared with others. This anonymity assisted the students in being truthful in sharing their views of mathematics because they know their views will not be connected back to them through the study. Additionally, I discussed with students that they have the right to withdraw from the study at any point without it affecting their grades in any way. During the class meeting I answered student questions in order to establish a trustworthy learning environment for my study.

I distributed and collected consent forms (Appendix A) in which the students’ parents gave me permission to have their children participate in the study. In the parent consent form, I explained the objectives of my study and

assured parents that it was their decision whether or not I could collect and analyze their child's data for the purpose of my study. In my consent letter I also asked parents to contact me if they have any additional questions or need further explanation of my study.

Prior to sending parent consent forms, I obtained my principal (see Appendix B) and assistant superintendent's consent to perform my action research on math communication at my school. She and I discussed how my study could best benefit the students' needs.

In addition I followed Arhar, Holly, and Kasten's (2001) ethical guidelines to assure that there will be no harm to my participants. In respecting the anonymity and privacy of my students I made sure that my field log was not accessible to students. It was locked in my filing cabinet in my classroom. I typed and printed out my field log observations only when students were out of the room (i.e. specials and after dismissal). I also obtained consent from the Moravian College Human Subjects Internal Review Board (HSIRB), a committee of individuals that examines study proposal to ensure the safety and anonymity of the participants.

Arhar, Holly, and Kasten (2001) describe triangulation as collecting data from a variety of perspectives in order to examine possible explanations for the data. In my study I triangulated my data by collecting data through a variety of methods. My data collection included the analysis of student entry samples that

spanned across the four month time period, student surveys, student informal interviews, field observations that include both participant observations as well as my reflections as the researcher. The informal student interviews provided me the opportunity to ensure that I had an accurate understanding of the students' perceptions. I wrote an average of two weekly observations in my field log to display the span of my observations throughout the length of the study. During my study I was consistent in writing each of my observations on the day it occurred. This ensured that my data was as detailed as possible to provide an accurate view of the events that occurred during that day's math communication class. In addition, by writing the entries in my field log soon after the time they occurred I was also able to quote student comments in my log. Initially I had intended to include pre and post test quantitative data in my study. However, after carefully analyzing the new district mandated assessments, I concluded that the tests did not present significant data for evaluating math communication.

I conducted my study for a four month period in order to collect sufficient data to show the progression in the students' entries in displaying their mathematical thinking progress.

In designing my study, I read numerous articles and studies to validate the strategies that I will be incorporating in my study. My researched also included articles on the importance of developing a students self –confidence with

mathematics in helping him/her be able to successfully communicate his/her mathematical thinking process.

I also discussed my study and findings with my research support group. As MacLean and Mohr (1999) and Arhar, Holly, and Kasten (2001) indicate a research study group can provide valuable feedback for a researcher to consider as he/she is conducting action research. Their feedback supplied me with multiple educators' viewpoints as I conducted my research. I was able to make needed modifications by incorporating their suggestions. Their feedback and suggestions assisted me in ensuring that my data and analysis were trustworthy and credible.

RESEARCH DESIGN AND METHODOLOGY

In my role as an action researcher I gathered data using a variety of qualitative methods. I utilized a field log as an organization tool to collect my data. By collecting data through participant observations, informal interviews, student work, and surveys I was able to analyze data through multiple perspectives.

Field Log

I used a field log to organize the data that I was collecting. As MacLean and Mohr (1999) indicated, a field log is a valuable tool that can provides action researchers with a systematic way of gathering data. I divided my field log into three sections: field notes, student work, and surveys. All the contents within each section were organized chronologically.

Field Notes: Participant Observations and Informal Interviews

The first section of my field log contained all my anecdotal notes on participant observations followed by my personal reflections. I wrote a participant observation following each of my math communication class periods. Within each of my participant observations I included the date, math topic, communication strategy, scaffolding method, types of math communication levels that were present, as well as student actions and comments during the class. In addition each participant observation includes observer comments. Bogdan and Biklen (1998) define observer comments as researcher's perceptions relating to what

he/she is observing. According to Ely, Vinz, Downing, and Anzul (2001), acknowledging observer comments assists in identifying potential researcher bias. They indicate that by bracketing off the observer comments, one can distinguish between the researcher's personal feelings and reflections from the actual events that take place.

Arhar, Holly, and Kasten (2001) describe informal interviews as a "purposeful conservation . . . to understand the perspective of the student" (p.152). As I worked with students in assisting them in thinking through their mathematical reasoning skills, I often asked students questions about their views of math communication as well as their perceptions on the progress that they were making. I documented these interviews within my participant observations, while others are included in my reflection notes, especially when the communication occurred during the third grade cross-grouping math time. Additionally, I also conducted informal interviews when I wanted to ask students to elaborate on a response that they had provided on one of the surveys that I conducted.

Student Work

An important component of my data collection focused on student work. The students' math communication binders created a portfolio of performance assessment tasks in which each student had the opportunity to write his/her mathematical thinking on a variety of topics within the mathematical field. Through the students' explanations of their mathematical reasoning skills in

completing a mathematical problem, I was able to chart and analyze each student's success in math communication. By analyzing the student work I was able to plan and modify upcoming lessons based on my students' current needs (Arhar, et al, 2001).

Surveys

MacLean and Mohr (1999) indicate that surveys present the researcher with a general awareness of the students' perceptions toward a study's topic. My initial survey provided me with the students' views of mathematics and how they perceive themselves as a student of mathematics. The initial survey assisted me in designing lessons that addressed the students' predispositions and apprehension towards mathematics as I started to present the notion of math communication. I conducted the second survey at the commencement of the third month of my study. Many of the questions were repeated from the initial survey because I wanted to determine how the students' views of math and themselves were changing. Due to the fact that the students were grouped per unit of instruction starting with chapter two, the questions that correlated to the math class in general were slightly changed to focus on math communication. Even though my initial plan was to conduct the survey again at the conclusion of my study, I felt that the informal interviews that I was conducting at that stage in my study were providing me with significant and sufficient data. Because of the consistency in

the data that I was able to gather from the students, the survey would have been repetitive to the students.

MY STORY

Creating a Positive Learning Environment

“ . . . acquiring an additional code comes from . . . connecting the language form with all that is self-affirming and esteem-building, inviting and fun” (Delpit, 2002, p. 39).

Delpit’s quote offers valuable insight into the environment that I carefully worked to create in my classroom. Even though the majority of my students have been exposed to American English as they grew up, mathematical thinking and communicating using mathematical terms can be a whole new language communication for them. Starting with the planning stage of my study I wanted to ensure that my classroom presented a nurturing learning environment in which my students would feel comfortable with the notion of “communicating” mathematically. As Delpit’s statement indicates, I would need to foster an environment in which the students would enjoy communicating mathematically and take ownership of the language as they find themselves comfortable with the math “code.”

I conducted my study, Incorporating Writing in Order to Better Assist Students in Understanding Their Mathematical Thinking Process, in a third grade regular education classroom. Our school is one of seven elementary buildings within a suburban-rural district. Our school, although considered suburban, lies

closest to an urban area. We have an increasing number of minority students with a variety of religious and socio-economic backgrounds. We are a Title 1 school.

Our class started out the year with nineteen students all of which have received parent consent to be included in the reporting and data of my study. Twelve of the students are girls, while seven are boys. One student, a girl, left our school in early November and therefore will not be included in the final data analysis for the purpose of this study. Three students are receiving additional in class math support through the District Remedial Program (DRP). Two students are receiving DRP support, mainly for problem solving, while one student receives support for all components within mathematics. One student is currently in the monitor stage for ESL, which means that he has exited the program; however, if needed, he can receive additional support through our ESL teacher. There are no students with Individualized Education Plans, IEP, that require any specific designed instruction.

The preparations for my study commenced well before the students arrived in my classroom in September. Along the top of the bulletin boards on the back wall, I organized my math word wall. I created posters that categorized the math terminology by math topics i.e. place value, geometry, capacity, weight, ect., and placed them on the back wall. In the middle of the wall I included the key words for deciphering between mathematical computations. Figure 1 shows the layout of posters for students to reference. Appendix C contains picture

samples of the individual math wall terminology posters. I hoped the math word wall would provide the students with much needed terminology at a quick eye's glance.

Figure 1. Math Word Wall. This photograph displays the layout of math communication posters in the classroom for students to utilize as a reference as needed.

In addition, I arranged the students' desks into groups, pods, of four desks each. I wanted the students to be exposed to cooperative learning groups from the very beginning of the school year. One additional task was for me to select a versatile tool to use for storing and organizing our math communication entries. I decided to purchase a half inch binder for each of my students. I wanted to ensure that we would be able to add to our binder any type of communication medium that we completed. I hoped the slipcover on the front of the binder would allow the students to add a personal touch to their math communication binders.

As I started my study once the students arrived in my classroom, my first goal was to learn as much as I could about my students and their perceptions of mathematics. Following the advice of Furner and Duffy (2002), I wanted to be able to address the students' feelings and conceptions of mathematics as I designed my lessons to assist them in expressing their mathematical thinking process through writing.

One of the ways that I learned about my students' views was by administering a survey. Recognizing that the first week of school in a new grade can be an intimidating experience for some students, I decided not to conduct my survey during this period. During the first week of school, similar to previous years, I worked on creating a positive learning environment where each student feels he/she is a part of the new family that is being established. I felt that this environment is key in order for students to feel comfortable in expressing themselves honestly, a key component to my study.

When I administered the survey (Appendix D) during the second week of school, I learned a lot of valuable information about my students. Starting with the results of the very first question, I enjoy math class; I discovered just how much my students differ in their view of math as a whole. The results of students' responses from the multiple choice section of the survey are included in the Table 1.

Table 1

Preliminary Student Surveys

	Never	Sometimes	Most of the Time	Always
I enjoy math class.	5	7	5	2
I am a good problem solver.	1	5	10	3
I wrote in math class last year.	1	8	6	4
I think math homework is easy.	1	9	2	6

More valuable than the circled responses to the four statements was the relationship between the students' open responses and how they perceived math in their responses to the four statements. For a majority of the students their responses were directly correlated to feelings of success or lack there of with mathematics. For example, Juliet circled "Always" for the first statement indicating she enjoyed math class. Underneath for her comment she wrote, " I enjoy math class because I'm good at it." Kloe who circled "Most of the Time" for the statement I am a good problem solver wrote "because I get a lot of problems right." On the other hand, Ryan who circled "Never" for the first statement, I enjoy math class, wrote, "It's not going to be easy." Mark who circled "Never" to the second statement, I am a good problem solver, wrote, "it's hard." An additional insight that I learned from my students was that two students indicated that they like the challenging part of mathematics. In his survey Brad indicated he always likes math because "it is challenging." Similarly,

Frances wrote, “I like it most of the time because there are hard problems.” The students’ comments on their apprehensions and successes with mathematics were a significant piece of how I would address math communication with my students.

My observations of the students during the first week of school were a valuable resource in continuing to learn about my students. I quickly learned that my students’ math abilities varied just as much as their perceptions about mathematics. As Vygotsky (1978) indicates, “A well known and empirically established fact is that learning should be matched in some manner with the child’s developmental level” (p. 85). As I began my study I viewed my biggest challenge as how was I going to meet each student’s developmental level in order to ensure that the learning environment continued to provide the student with the opportunity to feel successful with math communication.

At the class meeting where I explained the study to my students, they seemed amazed that I was still learning too, and as a result an additional connection between the students and me was created. Charles asked why I was going back to school. When I told him that I wanted to continue to learn how I could help my students even better, he just smiled and said, “Oh, so you’re learning just like us.” When I explained to the students that they teach me things everyday, the surprised look on Charles’s face and the rest of the students was alarming. The students couldn’t quite fathom the fact that I learn from them. As I continued to describe how my observations of students assist me in becoming a

better teacher, the students' facial expressions began to change from shocked to excited. Perhaps, some students were simply excited because they had their own math binder. Others, like Betty and Kate, however focused on the fact that I was going to school just like they do and continuously asked, "So when do we get to read your college report on us?"

Visual Communication – The Beginning

"This gives us grounds for regarding children's drawings as a preliminary stage in the development of written language" (Vygotsky, 1978, p. 113)

As the students gathered on the carpet with their eyes wide opened looking at the brainstorming web on the board titled "Why do we need math?" they didn't realize that today marked the commencement of my dream in assisting them in communicating their mathematical thinking. As the students volunteered some reasons as to why we need math, most of the responses were very general or impersonal such as Karen's answer "to have good grades," Kate's "so that we know how to add and subtract," or Rich's response "we don't really need math for anything." The students were not making real life connections for mathematics, a valuable insight in understanding the importance of math communication.

As I read the story *A Day With No Math,* a story that shares the life of a boy who wakes up one day and all evidence of math from the clock, to measurements, to speed limit had vanished, the students' facial expressions and laughs demonstrated that they knew the importance of specific math tools when

they were not present. For example, after I read that all the measurements for the ingredients were missing, I could see the surprised expression on Charles' face because he knew the breakfast was not going turn out okay. Before reading the second half of the book where the boy now awakes with all evidence of mathematic items back, I asked the students, "Why do we need math," according to this story. The quick response of hands raising, resulted in students sharing real-life math connection demonstrating the importance of math. At the conclusion of the story, I asked the students to draw a picture answering the question we started with today, "Why do we need math?" Figure 2 on page 34 displays one student's sample drawing of how she believes people use math in their everyday lives.

At the conclusion of class the students had the opportunity to each share their drawings with the people at their pods. I wanted to establish conversation within math communication from the start, and the comfort that I observed in the students in sharing their representations on the usefulness of math in everyday life demonstrated Freire's (1970) comment, "Without dialogue there is no communication, and without communication there can be no true education" (p. 92-93). The students were very open to sharing and acknowledging the positive attributes about each others' representation of math. I could see the closeness of the classroom environment effects every in the development of the study.

Figure 2. “Why We Need Math?” Demonstrates one student’s perspective on how math is used in everyday live.

The students’ drawings of how we need math in everyday life became their cover for their math binders. As we continued to discuss the purpose of math and the actual study that I would be conducting, the students’ excitement was apparent in different forms during the first few weeks. A few days after the students created their pictures they then moved on to general written math communication. The students were then asked to write a written response to “Why We Need Math?” After the students shared their written responses, I had them select a neat use of math that they hadn’t thought about until their classmate

share it to their own entry. Figure 3 shows one student's written communication sample of how she uses math. Appendix E contains a template for this sample of general math communication.

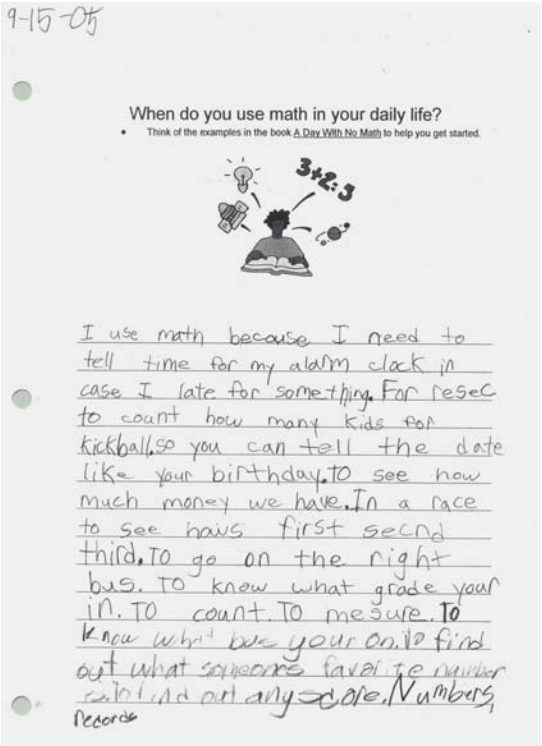


Figure 3. General Math Communication. A student's general math communication sample in which she shares the many ways she uses math in her everyday life.

Procedural Communication Begins

After three weeks of math self-esteem building and general math communication topics, the students were ready for procedural communication.

Sample templates of procedural communication can be found in Appendix F. In addition, during the first three weeks of school, I made it a point to ask students to orally explain why we did particular steps in every math topic that we were learning. By the third week in September we were learning how to order numbers from least to greatest or vise versa. As I had worked with the students I knew that some students could participate in a shared demonstration of ordering numbers while others were only ready for me to model to steps at this point. Therefore, I combined the two techniques in meeting the students' different developmental needs. I called on students to share their steps in ordering numbers. As I wrote the procedures on the overhead in placing the three numbers in order, I repeated the steps for the students who needed the additional support and guidance through more of a model designed instruction. We worked on incorporating math terminology, such as ones, tens, and hundreds into our explanations. As we concluded each of the steps, the students wrote each step on their own paper. Later the students completed their own problem on ordering numbers that was on the bottom of the paper. It became apparent at this stage how much the students ranged in their abilities. While some, such as Juliet and Kloe completed their problem with little assistance from the example, others like Betty and Karen needed the sample each step of the way. The positive observation that I made with these students was that they were motivated to use the sample to aid them. Rich, however, almost had a meltdown. As I was making my way towards his pod I

heard “I hate math. I don’t know what to do. I can’t do this.” Rich is the type of student who isn’t afraid to tell me on a daily basis he does not like math. He has shared with me a few times the statement “I can’t do math, so I don’t like math.” In addition by 2:15 PM, our mandatory scheduled math time, he had used up much of his daily attention span and struggled to stay focused. As I walked over to provide a more guided practice experience for Rich in today’s assignment, I knew that he had had some success in orally sharing math procedure during the previous week so I was prepared to work on transferring his oral success to hopefully written communication. When I pulled up a chair next to his desk, he seemed to have a sign of relief in his eyes as if he knew help had arrived. First I had him reread the sample from the previous day. I asked him to underline all the math words that we had used so that he would be able to reference them later. The following dramatization followed for the next few minutes in initiating Rich’s communication.

Teacher: Rich, let’s pretend we don’t have the communication lines. Out of the three numbers which one is the least?

Rich: 352.

Teacher: How do you know 352 is the least?

Rich: Because it only has three small numbers.

Teacher: Rich, look at the sample from the other day. Can you find the word that names the individual numbers because 352 is one number?

Rich: You mean digits. 352 has the fewest digits. Oh, but 635 has three digits, too.

Teacher: You're right! So how do you know 352 is smaller?

Rich: Because the three is smaller than the six.

Teacher: Correct. Why did you compare the three and the six?

Rich: It has bigger digits.

Teacher: Where does it have bigger digits?

Rich: In the front. So that makes it greater.

Teacher: Rich can you look at the sample from the other day and find the math word that names that front place?

Rich: I can't find anything.

Teacher: Let's look at our math word wall on the place value section, what words are there that we can possibly use?

Rich: Ones, tens, hundreds, thousands.

Teacher: Rich, on the numbers in our problem, which of those words describes the front?

Rich: The hundreds.

Teacher: Awesome. So why is 352 smaller?

Rich: Because it has a smaller digit in the front.

Teacher: What is that front place called?

Rich: The hundreds. Oh, because it has a smaller digit in the hundreds.

Teacher: Awesome!! I knew you could do it. Alright, we are now ready to write the first part of the problem. Let's write the smallest number on the answer line and then write what you told me makes that number the smallest.

As Rich worked on his explanation, I walked around the room to assist other students and provide feedback as needed. When I made by way back to his desk, he already had written the last two numbers in order. As I sat next to him, I could tell from the excited look on his face as well as his eagerness to close the math binder that he thought he was all done.

Teacher: Rich, can I take a look at what you came up with?

Rich hesitantly opens his binder back up.

Teacher: Hey! You did a super job figuring out the last two numbers.

Rich: That was easy, the writing is not! Do I have to write this part, too.

Teacher: Well, let's talk about how you knew what number came next.

Rich: 635 is bigger than 28,896. So I wrote that one in the middle.

Teacher: How do you know that 28,896 is bigger?

Rich: It has more digits.

Teacher: Super, that's what you need to write down!

Throughout the beginning stages of the study, I worked on building Rich's self-confidence in helping him realize that he can write about his mathematical thinking. As Delpit (2002) states, "Those of us who teach must first make our

students recognize their potential brilliance" (p. 46). I have to hope that with time Rich realizes that I will not let him give up, and that he can to it even if he does need a little assistance at first.

Rich like many students benefited from oral communication in which they verbally talk through the needed steps in communicating their mathematical reasoning skills. As I posed prompting questions to guide the students through their thinking process, I wanted to ensure that the students arrive at the answer on their own terms. I often suggested that they use references, such as the math wall as well as previous samples in their math communication binder, in order to assist them in discovering the process that is taking place. My main goal was to assist the students in uncovering the mental process that automatically transpires in their heads, which many times they, themselves don't realize occurs. Delpit (2002) states "Teacher talk and student talk are essential components that determine the quality of learning in the classroom" (p. 147). As I conferred with students while they complete their communication tasks, I wanted to ensure that our dialogue assisted the students in developing their thinking process.

First Signs of Students "Living" Math Communication

The following is an insert from my field log on October 4, 2005, when I came to the realization that all the whys that I have asked in math class are starting to have an effect in the students' ability to communicate their mathematical process.

Our math class today focused on problem solving skills with an emphasis on identifying strategies to use. I was looking forward to seeing how the students were going to adapt our current communication techniques into the problem solving strategies. Even though I didn't particularly intend to focus on communication with today's topic because it was only the second day we were focusing on problem solving, I wanted to create a mental baseline of where the students are. One of the exercises had the students selecting a strategy to solve a problem. I then had three students, Juliet, Nicole, and Emmy, show their strategy on the board. Each student then orally described his or her strategy to the class. I was impressed how well the students did in describing their thinking process. Even though they did not integrate the correct math vocabulary into their explanations, they were able to share their thoughts in sequential order. One of my focuses was to point out to the students that each of the strategies was valid. Juliet actually used a combination of the picture and table strategy and it was neat to point out that the book strategies were not the only way to go. Two of my low students, Rachel and Betty, did not know how to start the problem. But when they listened to the explanation, I told them to select the strategy that made the most sense to use in solving the problem. I later went to each of them and asked them to tell me how they were able to solve the problem, and with some sequence word

prompting they were able to describe the process of the strategy they used. It was nice to see that they were able to use a previous students' information and adapt it to their level.

Later in the class, the students were independently solving some word problems when Karen gave me the second validation that the students are "living" math communication. One of the word problems on the worksheet asked the students to provide the problem's answer in a complete sentence; however, when I went over to her she was explaining the strategy she used to solve the problem. I had her reread the question; and she then commented that she is so used to having to describe what she did that she right away thought that was what the question asked. While I had her correct her answer, according to the directions, I pointed out that she was ahead of the game because she was ready to communicate her solution method.

I discovered that the students were taking their own initiative to math communication. They weren't hesitant to share their personal strategies, and they were able to recognize how others used a strategy that they didn't think of. I was pleased of the progress that the students were making and how receptive they were to communicate math concepts.

Abstract Communication -- Take Two!

As we neared the end of the first week in October, I was extremely pleased with the progress the students were making in being able to explain their steps in solving math problems. It had become a regular routine for students to explain their procedures along with their answers when sharing in math class. Most of the students didn't even need me to ask, "How did you arrive at that answer?" Maybe it was the students' success or possibly my own over confidence in their skills, but I soon discovered that even when the students are showing remarkable progress it is essential to continue to follow the teaching strategies when introducing a new level of communication.

One of the topics in the first chapter was problem solving, specifically the steps one should take in solving word problems. The students explored the strategies the textbook recommended such as reading the question, determining what the problem is asking, finding the important information that is needed within the problem, solving the problem, checking one's work, writing the answer, and making sure that the answer makes sense to the problem's question. Problem solving unlike most of the other concepts in chapter one is a very abstract concept. When I assigned a math communication assignment with the question "What have you learned about problem solving?" I received very mixed responses. As I conferred with students while they were completing their task I realized that most students were not ready to complete this task independently.

While most responses had some signs of problem solving, many writings were hard to follow and out of sequence. For the first time the students had not met the expectations that I was accustomed to. As I reflected on that day's math class, I learned an important lesson. Regardless, of how successful the students are at completing current tasks, when I ask them to complete a communication task that is at a higher level, in this case moving from the concrete topics of place value to the abstract topic of problem solving, I cannot assume that they are ready for this topic at a guided practice or independent level. Even though we discussed the question and a few students volunteered some things that they learned to do when solving a word problem, I asked the students to complete this task mostly at an independent level. Something they were not ready to complete in writing, yet.

Two days later, however, the scene in our class was very different. After brainstorming the steps in solving word problems as a class, each student received a paper that included each of the steps. The trick was that the steps were not in correct order. The students' mission was to cut out the steps and then glue them in order on the second sheet that they received. Appendix G contains the templates for this hands-on problem solving communication activity. The anticipation on Sam and Brad's faces displayed that they couldn't wait to complete their mission. A much difference expression than I had received from the two students on the previous day, when they couldn't think of what to write. As I was working with Rachel in assisting her manipulate the steps by questioning what she does and

which step describes the same thing, Michelle and Emmy came rushing over and together said, “We found different ways to put the steps in order! Can you check if they are both right?” Emmy continued, “The beginning steps are the same, but the last ones are different.” I proceeded to ask her, “Does everyone solve each problem the exactly same way?” Michelle was the one that quickly responded, “Of course, not!” I asked both of the students to share their steps. At the conclusion I asked both if the order of their last steps made sense. Both responded that the steps did make sense. I asked them to look at each of their steps and see if they could discover what is different and why they were both right.

As they returned to their pod, I continued to help Rachel. She now had more initiative in manipulating her problem solving steps to put the pieces in order. I commented to Rachel that she was doing a good job. She said something that showed me the power of student sharing. She stated, “I want to see if I can find a different way, too!” I was surprised to see Rachel, a student who often struggles in math, was motivated simply by seeing her classmates’ success and wanted to imitate their success. As I continued to walk to the different pods, I saw that a lot of the students were working together. Although I didn’t initially plan to have to students cooperatively complete this task, I was pleased to see the students openly work together and assisting each other. I heard comments similar to, “No you can’t do that yet, you have to figure out the question first,” or “That way works, too.” When I came upon Emmy and Michelle’s table, their smiling

faces were a sign that they were eager to share why they were both correct. Figure 4 shows two students of how the steps could be placed in a different order.

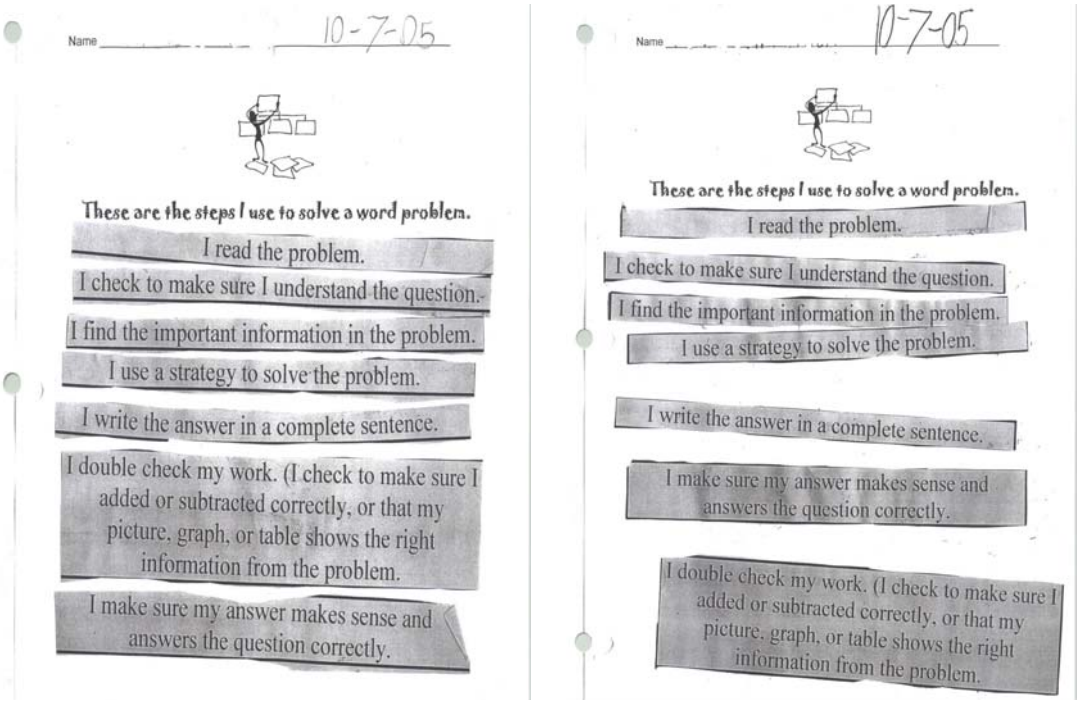


Figure 4. Hands-On Problem Solving Communication. Illustrates two student samples indicating each student's problem solving steps.

As they shared why the last steps could be in different order, I reflected on the difference of the two classes; both of which focused on communicating math problem solving. What a difference a visual instead of writing provided the students! The cycle of manipulating the steps gave the students an opportunity to

take an active role in their communication. I also learned that my students aren't afraid of taking the initiative of working together and openly share their perspectives.

Time, Where Do I Find It?

Ryan and Rillero (1996) as well as Burn (2001) address time constrains when conducting writing communication in mathematics. They indicate that writing prompts can be a useful way to either start or end math class as a review of concepts. However, I quickly discovered that at the beginning stages of introducing writing into math communication, third graders need a lot more time than simply a few minutes at the beginning or end of class. A couple times during the first month of the study, I had to modify my math lesson plans to adjust for the time the students needed to complete a math communication activity. But I didn't stress when making lesson modifications because I could see the students were learning the chapter concepts to a new level than I saw in the students in the previous years. It wasn't until chapter two when the students became cross-grouped for math that time became a major problem in my study. For the first time this year in our school district third grade joined fourth and fifth grade in cross-grouping students for math instruction. Prior to each chapter, the students complete a pretest examining their prior knowledge of that chapter's specific concepts. Based on the scores as well as teacher input, the students are places in the high, average, or low group. Facing this issue, I now needed to find a time in

my schedule to dedicate to math communication for my homeroom students. It would have been much easier to complete this study with my math classes. However, the fact that I would have different students within each chapter's class, I would not have been able to create a constant continuum to analyze the students' progress.

During a three week period of time, I faced a constant battle with my schedule. Originally I scheduled math communication three days in a ten-minute block right before math class. I thought this would be a great warm-up activity as they prepare to go to math class. However, ten minutes was not a sufficient amount of time for both a mini-lesson as well as students having the opportunity to think through their process and write it down. This ten- minute period of time brought frustration to both students and me. During the first week, the students expressed that they needed more time to write. Even though I comforted them by telling them they could finish it the next day, they were in a flow and wanted to finish their writing. Their responses started becoming fragmented and were not showing the same level of completion. As I pondered the problem, I ended up not having math communication for a few days as I played with the schedule. I did not want the frustration and time restriction to change the safe environment that had been created. I didn't realize the connection that students were having with their math communication until Brad one day asked, "When are we writing in our math binders again? That was fun. Do we have to be in your math class to write in

them?" Juliet followed by saying, "Were we not doing a good job? Did you stop your school project?" At this point I realized that I needed to share with the students what was happening with their math communication. I explained to students that they are doing a great job, and we did not stop writing in our math communication books. I explained to them that as I saw them struggling and getting frustrated with not having enough time to complete their tasks, I did not want them to rush. I told them that it was my job to find a time in our schedule that would allow them to be able to complete the math communication task as we were doing them before we started to switch classes for math. Brad with a smile raised his hand. After I called his name he said, "Can we just stay with you for math? Then we can have fun writing again, and you don't have to worry about finding a new time." As he finished all I heard was a chorus of "Yes, we should do that!" from the students. As flattered as I was that the students wanted to keep writing, I explained to them how math grouping helps them learn math in a more effective way based on what they already know. I promised them that I would figure out a way by the next day how we would schedule our days to make sure we have math communication. As I have read studies on incorporating writing in math communication, researchers often face tribulations and disappointments with time constraints, and I was no different. What I found inspiring was that my students provided me with the motivation to further scrutinize my schedule in

finding time for math communication. The following poem reveals my conflict with finding time as I analyzed my schedule from every vantage point.

Poem: Time

Time

Time, where do I find it?

Students thinking, sharing, writing math.

How can I give them time?

Ten minutes is just not enough!!!

Two or three days how much time can I find?

Reading, Writing, Spelling, nothing can go?

Science, Social Studies, It all must stay!

If I could only buy time?

The next day I shared with the students my finding and new schedule changes. As I announced to the class during morning announcements that I had a plan, their looks of anticipation revealed that they knew exactly what I talking about. Kate and Nicole even started taking out their math binders without me even revealing that the plan was for scheduling math communication. I told the students that I think I finally found the answer!! I announced that I changed handwriting to the time block right before math, which means that we would only be doing one letter at a time instead of the two we were currently learning. I calmed their fears by revealing that we would still learn four letters a week because we have four ten

minute time slots set aside, and we would be ready to write in cursive by the end of January. After all, I couldn't take the anticipated cursive away from the students either. I continued by telling the students that we would then use the two twenty-five minute periods right after lunch that were originally scheduled for handwriting for our math communication. The looks on the students' faces were all the thanks that I needed for the late night in struggling with time. But the most satisfying was Charles' comment, "Cool, that means we are going to have it today because we have handwriting on day two!!!"

Students Organize their Mathematical Thinking

The next three weeks brought organization to the students' writings of their mathematical thinking progress. As we would soon be encountering topics that would require additional writing in showing the process in solving the problems, the next step was to help the students develop a way to write their thinking progress in an easy to read fashion. On the first day that I introduced the t-chart, I explained to the students that this chart looks exactly like its name. Then on top left of the horizontal line I wrote "Problem" and on the right side I wrote "Explanation." I explained to the students that on the left side we would complete the actual computations involved in a problem and on the right side we would right what we were doing. I continued by modeling an example problem of how I could use the t-chart to explain my steps in an addition problem. After I modeled how to use a t-chart, the students were eager to create their own t-chart. Figure 5

shows the t-chart format. The students surprised me with the ease that they transferred our past organization of writing to the t-chart in their first attempts. While many used the sample on the board as a guide, I noticed that a few, Juliet, Angel, and Brad, completed this task completely independently. I was eager to determine if their ease in adapting to the t-chart would continue as I presented further suggestions for them to incorporate in their math communication.

Problem	**Explanation**

Figure 5. T-chart. An organizational tool for students to separate their work and written explanation of their mathematical thinking in completing communication assignments.

The Power of Self- Reflection and Assessment

Burns (2001) pointed out the importance of having the students share their writings with each other and having open discussions about the writings followed by a time for revisions in which students could modify and/or add to their responses in order to "Boost real learning" (p.18). Burns indicates that this would provide the students with an opportunity to see others writings and thinking

progress and adapt some of their steps to themselves. It would also provide the struggling students with an additional sample. I used Burns's model in assisting students in developing their writings of their mathematical thinking progress throughout the use of the t-chart. The two day per week time slots fit perfectly into this plan. On the first day I would complete a shared demonstration with the new topic and strategy for the first ten minutes. Then the students would have an opportunity to complete a t-chart on a similar task. On the second day I had overheads of student samples. We would analyze the samples and identify the positives, or things that this person did well, and the things to work on, the items this person either forgot to include or can explain a little better because it was not easy to follow. Then the student would assess their own writings by identifying the positives that they did correctly and things they needed to work on in the future. Figure 6 on page 54 demonstrates one student's reflection and how she modified her entry to include the needed items.

One of the focal moments in the study came during the third week of using t-charts, when I noticed that the students were starting to read their personal assessments prior to starting a new task. During the third week I modeled how the importance of using math terminology in the explanations. For some reason, on this day I hadn't asked the students to share what they identified as the items that they were going to work on next time. As I walked around the room after

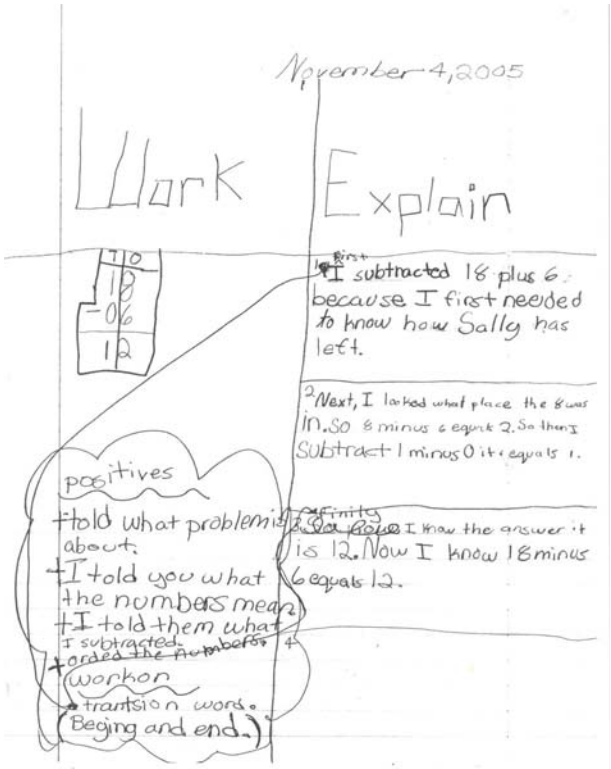


Figure 6. Student Sample - Personal Reflection. Demonstrates one student's self-reflections and personal revisions on a procedural math communication task.

completing a shared demonstration of the t-chart subtraction problems, I noticed Nicole, Emmy, Michelle, and Brad reading their t-charts from the previous week. When I walked over to Nicole, I asked her why she was looking back; she responded that she was looking at the transition words that she needed to use this time. At this moment I realized that students were independently using their reflections. I quickly shared this amazing feat with the students and how proud I was that Nicole was rereading her "positives and things I need to work on"

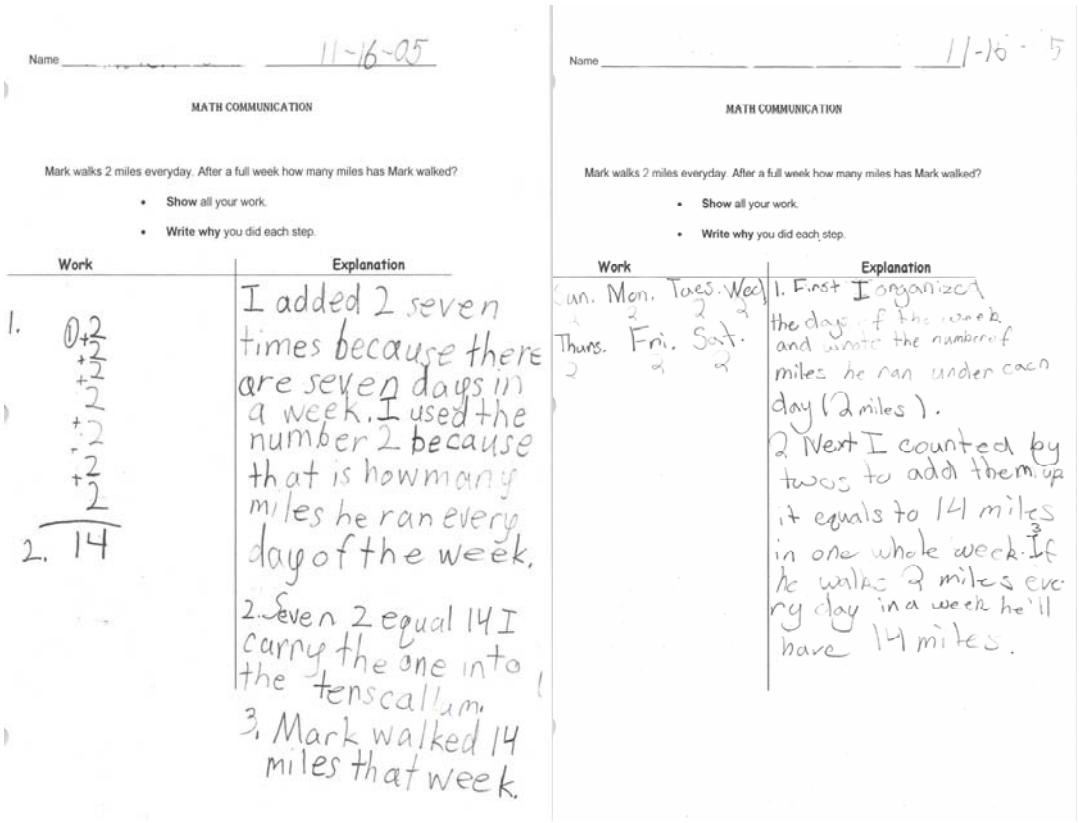
section from the previous entry and checking over today's entry. As I explained to the students what I had seen, I soon saw others utilizing the same strategy. But this was not the best part. During the upcoming weeks, I continued to witness students independently rereading their reflections and modifying entries in order to improve their writings of their mathematical thinking process in the various tasks.

Sharing student samples on overheads brought more than an opportunity for students to self-assess and reflect based on their peer's writings, it provided motivation for the students to do their best. Originally I was concerned about sharing student samples for two reasons. First, I did not know if students were going to be able to openly evaluate each other's samples. Second, and perhaps most important, I did not want the students to feel self-conscious in sharing their samples and having classmates evaluate their work. I did not want to damage the cooperative learning environment that we had created in our classroom that was proving so be so positive for the students. I was surprised by the openness the students had to have their writing on the overhead. It soon became evident everyone wanted their writing on the overhead. Even Rich was motivated to write!!!!!!!!!!!!!!!!! Rich, the student that I needed to motivate to start his writing each and everyday of math communication. Rich the king of the phrases "I can't do it! Why do we have to do this? You know I don't like math," was now independently writing in math communication. Why you may ask? As I

commented on how proud I was for him getting to work, he enthusiastically asked, “Can you pick my writing for the overhead?” It all became clear, then, and I quickly decided that this was the hook I needed to assist Rich in improving his writings. I shared with Rich that I select writings that are complete, neat, and included the strategy we are working on.

Sharing student samples on the overhead also allowed students the opportunity to analyze how problems can be solved in different ways. Too many times I have observed that students believe there is only one way to solve a particular problem, and they get themselves all worked up when they can’t think of how to go about solving the problem. I feel it is imperative for students to know that they can each solve problems using different strategies and still obtain the right answer. An important focus in sharing student samples was to cooperatively analyze how students used different strategies to all arrive at the same answer. Figure 7 on page 57 shows two student samples that we analyzed as a class in explaining how the two students used different methods to solve the same problem. By sharing student samples in which the students used different procedures, students were able to first hand witness how they could themselves solve problems in their own way. The students’ explanations revealed how each one of them started the problem with the part that they felt comfortable with and then went on from there. This helped the students feel more at ease when solving

word problems. In addition on some tasks I witnesses some students, like Frances, attempting to discover different ways to solve a particular problem.



Name 11-16-05

MATH COMMUNICATION

Mark walks 2 miles everyday. After a full week how many miles has Mark walked?

- **Show** all your work.
- **Write why** you did each step.

Work	Explanation
1. 0+2 +2 +2 +2 +2 +2 +2 2. 14	I added 2 seven times because there are seven days in a week. I used the number 2 because that is how many miles he ran every day of the week. 2. Seven 2 equal 14 I carry the one into the tens callum. 3. Mark walked 14 miles that week.

Name 11-16-05

MATH COMMUNICATION

Mark walks 2 miles everyday. After a full week how many miles has Mark walked?

- **Show** all your work.
- **Write why** you did each step.

Work	Explanation
Sun. Mon. Tues. Wed. 2 2 2 2 Thurs. Fri. Sat. 2 2 2	1. First I organized the days of the week and wrote the number of miles he ran under each day (2 miles). 2 Next I counted by twos to add them up it equals to 14 miles in one whole week. 3 If he walks 2 miles every day in a week he'll have 14 miles.

Figure 7. Student Samples - Problem Solving with Different. Illustrates how two students completed the same problem solving task using different thinking processes.

Student Surveys Bring Differencing Views

As I approached the middle of my study, I conducted a mid-study survey (Appendix H) to examine my students' views of math communication. I wanted to identify if my observations were consistent with what the students felt was

occurring. The survey was very similar to the initial survey because I also wanted to see how the student attitudes towards math have changed and/or remained the same. One of the differences was that I changed the questions that referred to math class to math communication because of having to slot math communication to a different part of the day due to math grouping. I also deleted the third statement, I wrote in math class last year, because that would not have been impacted since the initial survey. An additional question that I asked was “Why do you think we spent time on math communication.” I wanted to see if the students were able to share why communicating our math thinking process is important. I also changed the statements “I think math homework is easy” to “I understand my math homework.” I felt the word understand would provide me with a better indication if math communication was assisting the students in understanding their math concepts when working independently on their homework. Table 2 shows the results from the student surveys.

Table 2

Mid-Study Student Survey

	Never	Sometimes	Most of the Time	Always
I enjoy math communication.	5	7	5	2
I am a good problem solver.	1	5	10	3
I wrote in math class last year.	1	8	6	4
I think math homework is easy.	1	9	2	6

As I analyzed the surveys, I was optimist in how many more students have a positive attitude towards math as well as understanding their math homework. But similar to the initial survey most of the valuable information came from the student comments. The first noticeable change that I encountered was how many more students offered comments on the top section of this survey. They openly explained their choices in their first three statements, something many did not do in the original survey. These comments meant two things for me. First, the classroom environment continues to be more welcoming and the students are more comfortable expressing their comments. Secondly, the writing in math communication has provided that students with a means to share their thoughts!

The following pastiche gives voice to the continuum of my students' comments that they share in the survey.

Pastiche: Student Survey Voices

When I think of math communication I think of

- *a new way to find out a problem (*Brad)
- *having some fun* (Sam, Karen) *good stuff* (Mark) *it being fun* (Sam)
- *something new to LEARN* (Jacky)
- *my head's going to explode* (Rachel)

My favorite part of math communication is

- *the t-chart* (Brad, Ryan, Emmy, Charles, Michelle, Mark)
- *adding with regrouping* (Michelle)
- *solving problems* (Kloe, Frances)
- *when we glued the steps* (Betty)

My least part of math communication is

- *figuring out word problems* (Michelle, Sam)
- *trying to remember the steps* (Ryan)
- *making the t-chart* (Rachel) *when we did the t-chart I did not understand them* (Jacky)
- *explaining why I did* (Frances) *when it says explain* (Charles)

Why do you think we spend time on math communication?

- *to get smarter in math* (Michelle) *to do more math* (Brad)
- *because the teacher Mrs. Hillard loves writing in math* (Charles)
- *to be better in math when I grow up* (Ryan) *to help us in the future* (Michelle)
- *to show what we know and it involves math* (Angel) *to learn and understand math* (Jacky) *because it is a great learning process* (Kate)

As I analyzed the students' comments a few things were evident. First, I was able to confirm the students that I knew were struggling with math communication based on my observations. In addition, I also discovered two students that were having more difficulty than I realized. From my experiences in providing Rachel with one-on-one assistance, I knew that she was having a hard time explaining her thinking process. But her comment that math communication makes her head feel like it's "going to explode," reveals just how she's feeling. As I continued to work with Rachel it was important for me to ensure that I provide her with assistance before she gets to that struggling stage. The second student that alarmed me was Jacky. From her survey I learned that she did not

completely understand the t-chart, which we had been using for two weeks prior to the study. On the next few occasions I made sure that I met with Jacky more frequently as I worked with her in understanding the components of the t-chart and how to use it in organizing her thoughts. Even though a Jacky struggled with the t-chart for the most part the students responded positively to the t-chart and math communication. One of the interesting things that came through the study was Charles' comment on the fact that I love Math. As soon as he wrote the comment he called me over to show it to me. His facial expression revealed that he knew that wasn't all of it, but he wanted me to know that he knows my love of math. Even though I try to display my enthusiasm towards math, it wasn't until I read the student's comment and reflected back on the students' statements during math communication as well as in math class that I realized how much the students counted on that.

Peer Performance Assessment

As the students continued to improve their organization and math terminology, I observed that some students were still having difficulty in communicating each of their steps in their written explanation of their thinking process. Often times they would complete a step mentally and not even realize the importance of sharing that procedure in their writing. Even though the students were able to identify these “missing” steps when I asked them to explain how they had obtained specific numbers in our conferences, I wanted to assist the students in recognizing the importance of communicating each of their steps in solving problems. Through my past experiences, students have often benefited from actively performing a task as they write the directions for completing the actions. Initially, I was under the impression that as the students completed the task on the left- handed side of the t-chart they would automatically write the steps in the right side, and they were -- that was the problem! The students were explaining the steps that they acknowledge they perform and were omitting the mental steps that don’t realize they unconsciously completed. But after carefully observing and asking questions while the students completed their entries, I discovered the obstacle that the students were facing. The following dialogue between Kate, a student who often struggles with math concepts, and myself, helped me realize that some steps are so innate in the students that they don’t realize that they are

doing them. While completing a math regrouping problem 567 + 457, I asked Kate why she placed a one on top of the tens column after adding seven plus seven? She responded, “Well seven plus seven is fourteen and that’s where I put the one.” I then proceeded to ask, “Why?” Kate continued, “Well, the four in fourteen goes in the ones column and the one has to go there.” I asked, “Why didn’t you put it in the hundreds column above the five?” Kate responded, “Don’t you know that you always have to regroup to the next place value. You can’t skip over.” “Do you think students who are just learning to regroup would know where to put the one you regrouped?” I asked Kate. “Probably not. Oh, I get it. I should write that in my explanation of what I did.” As I reflected on the conversation that I had with Kate, I knew I needed to design a different way for the students to perform the task in order to discover the steps that they automatically are conducting mentally.

As I pondered how to assist the students in recognizing these steps that they automatically completed mentally, I decided to integrate a writing workshop strategy that I use when writing pieces that describe how to do a particular task. I often have the students read their steps as a classmate performs the task. Their partner is instructed to only perform the actions that are read. From my experience, this exercise often helps the writer acknowledge steps that he/she forgot to write. So I decided to try this strategy with math communication. The following play contains comments and actions from my field notes as I introduced

peer checking to the students and then had them complete an exercise using this strategy. As I modeled peer checking to students the importance of the partners' role in only completing the steps that were being described.

Play: Peer Performance Assessment

SCENE: *The students are sitting in three rows on the carpet, the normal carpet procedure. Brad is raising his hand.*

MRS. HILLARD: Yes, Brad.

BRAD: Are we going to make a t-chart today?

MRS. HILLARD: We will later, but I have a new strategy to share today for us to use with our math explanations.

(Brad smiles as he hears that the teacher state that they will complete a t-chart today.

I will need a volunteer that thinks he/she can carefully follow the directions that I read.

(About half of the students raise their hand.)

MRS. HILLARD: Betty come on up! *(Teacher selected a student that has displayed that math communication is difficult in order to demonstrate a realistic procedure with potential issues that may rise.)* Betty's job is going to complete the work side of my t-chart as I read the explanation that I wrote on my paper. *(Mrs. Hillard shows her writing to the students.)* The important part is that she

can not write anything that I don't read. I left of some things from my explanation and I want to see if you can find what I forgot to write.

(Betty picks up a piece of chalk and gets ready!)

MRS. HILLARD: Those of you at your seats should raise your hand if you think I forgot something in the steps as I read them. Betty please write the problem 387+ 123 on the board so that we can get started.

(Betty eagerly writes the problem.)

MRS. HILLARD: First, I added my ones seven plus three and got ten. I write the zero in the ones column. Next, I added eight plus two plus one

(Nearly all the students raised their hands.)

BETTY: Where did you get the one from?

RICH: *(Who calls out with excitement.)* I know! That was from seven plus three!

BETTY: But Mrs. Hillard did not tell me to write it down.

MRS. HILLARD: That's right! You found the first thing that I forgot to write down in my explanation. Let me add the regrouping to the explanation and then we can keep going. *(Writes the regrouping section in the paper.)* Let's see here. Then I added eight plus two plus one and got eleven. Finally, I added . . .

(Students raise their hands and Betty looks at teacher with a look indicating that she forgot something again.) Frances, you're raising your hand. What did I forget?

FRANCES: You added the tens, but you did not tell Betty where to write your answer.

TEACHER: I'm sure she would know where the digits go. I said I had a sum of eleven.

FRANCES: You tell us we have to explain everything, plus Betty isn't writing it down in her problem because she's following your directions.

TEACHER: *(Asks the class as if she was puzzled.)* Don't we already know where to write the digit?

(Many students nod affirming the answer, but raise their hands smiling.)

TEACHER: Karen, what's wrong?

KAREN: Remember the explanation teaches someone all your steps. You are supposed to think that they don't know anything about what you are solving.

TEACHER: *(Smiling)* I guess you got me!!! Sometimes we do some things so fast, that we forget to include in our writing the things that we can do without thinking. Let me add this step to my explanation and let's see if I wrote my last step correctly. After I got eleven, I wrote a one in the tens column answer section and a one on top of the hundreds column. Finally I added three plus one plus one and got five. I write down the five in the hundreds column. Finally, I discovered my answer is 510.

(The students start clapping.)

MRS. HILLARD: It looks like I finally got it! Now you are going to try this strategy! First, I will team you with a partner. You will then need to select who's going to be letter A and who's B. The A's will be completing a t-chart on 458+152, and the B's will use the problem 325+157. First, you complete the t-chart for your own partner away from your partner. Then get together with your partner and take turns in completing the same thing Betty and I did. Remember when you solve your partner's problem you can only do what they say.

(*Students complete their t-chart communication on their addition problem. The teacher meets with students as needed to assist prompt questions. As students complete their t-chart they start the exercise with their partners.)*

MICHELLE: *(Walking rapidly towards the teacher's direction)* Mrs. Hillard, I forgot something that Karen is not writing down. Can I add it to my explanation?

TEACHER: Was I able to added missing steps to my writing?

MICHELLE: Yes, does that mean I can too?

TEACHER: You got it! This strategy is to help us with our explanations so we can add anything we forgot.

MICHELLE: Thanks. *(Smiling and enthusiastically writing things down as she walks towards her partner).*

TEACHER: Remember class as you complete the checking with your partner, you can write down any step you forgot just like I did!

(Students work on peer checking exercise.)

ANGEL: I liked this checking!! *(As he walks to his seat after completing the activity with his partner.)*

TEACHER: *(As students have returned to their desks after completing the exercise.)* What does today's activity remind us of our math communication? *(Numerous students raise their hands eagerly!)*

TEACHER: Yes, Mark?

MARK: We have to explain all the steps, even the easy ones! You need to think that the other person who reads it does not know anything.

TEACHER: Good job. Next time remember to carefully explain all the steps even the ones that you do mentally. Those steps are just as important. Yes, Rachel?

RACHEL: Are we going to do this again?

TEACHER: As long as it helps us. Rachel, how did this help you?

RACHEL: Well, when my partner was doing the things I read, I could tell what I forgot and that helps.

TEACHER: Wonderful!! It looks like you got it!

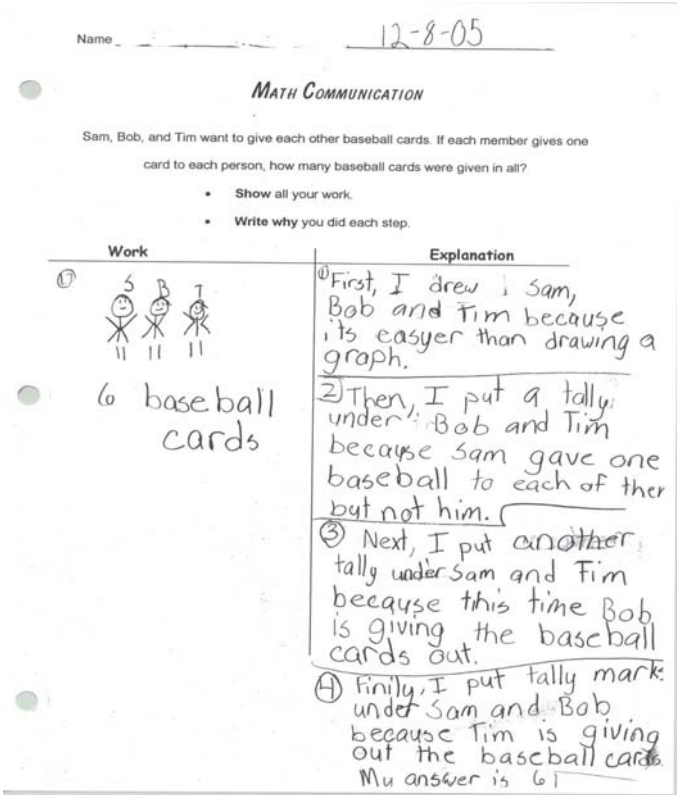
As I reflected on the students' performance with peer checking, especially how they discovered the "missing" steps in their explanations, I hoped the success I saw on the first day would continue. As the students completed peer checking exercises in subsequent weeks, I witnessed that their writings were becoming more in depth.

Struggling Students Find Success: While Others Make Connections

As the time period of my study started coming to an end in December, I reread student entries, conducted informal interviews, as well as analyzed my field notes in assessing student progress. One of the days I conducted an informal interview with Rachel, Karen, and Betty who were working on a subtraction with regrouping communication problem. I witnessed that they were conducting a peer checking in double-checking to see if they had all the steps. Two of the three students, Betty and Rachel, I had devoted significant time in modeling and providing one-on-one assistance and Karen a hard working student whom academics don't come naturally were independently working on a strategy that seems to work for all three of them. As I talked with the three girls, Rachel revealed that "this wasn't so bad. I can do it now." As I walked away I heard Karen, "Let's see if we have our math words." I walked away smiling to meet with the next group.

Rich keeps asking when his work is going to be on the overhead. Each time Rich and I look at one section of the communication: completeness, neatness, organization, and math terminology, which he continues to work on for next time. As I continued to pick student samples for us to analyze, I knew progress had been made when Jacky was one of the samples that I selected. Not only was it Jacky's sample, a student who was confused about t-chart and did not understand them, but it was also a problem solving sample, a task Jacky often

struggles with. Figure 8 displays Jacky's growth in using a t-chart to communication her math thinking processing in problem solving.



Name 12-8-05

MATH COMMUNICATION

Sam, Bob, and Tim want to give each other baseball cards. If each member gives one card to each person, how many baseball cards were given in all?

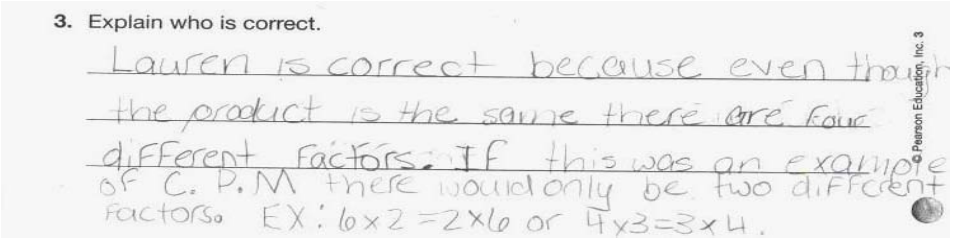
- **Show** all your work.
- **Write why** you did each step.

Work	Explanation
① S B T 6 baseball cards	①First, I drew Sam, Bob and Tim because its easyer than drawing a graph. ②Then, I put a tally under Bob and Tim because Sam gave one baseball to each of ther but not him. ③ Next, I put another tally under Sam and Tim because this time Bob is giving the baseball cards out. ④ Finily, I put tally mark: under Sam and Bob because Tim is giving out the baseball cards. Mu answer is 6!

Figure 8. Jacky's Problem Solving T-chart. Student sample that demonstrates inferential problem solving communication.

As struggling students started to find success in communicating their mathematical thinking through the use of t-charts, others commenced to make connections to math assignments. While teaching the different math communication strategies, I could only hope and guide the students to make the

needed connections in transferring what they learn to their daily work. I remember asking myself, “Will they use the math key words during math class and assignments? Will they continue to organize their thinking process when out of the math communication venue?” I can still picture Kate’s proud smile as she walked into class after math and explained how her math teacher commended her on how well she used the words sum and addends in her description. This was my first sign that the students were transferring what they were learning to context. As my grade level colleagues continued inform me on my students’ were using math terminology in their class discussions and assignments, I started being optimistic in my students’ progress with math communication. As I read three students’, all of which were in my the top math class for chapter 5, specific detailed explanations shared in Figure 9, I knew that the students were on their way to finding success in math communication!


3. Explain who is correct.

Lauren is correct because even though the product is the same there are four different factors. If this was an example of C. P. M there would only be two different factors. EX: 6x2 = 2x6 or 4x3=3x4.

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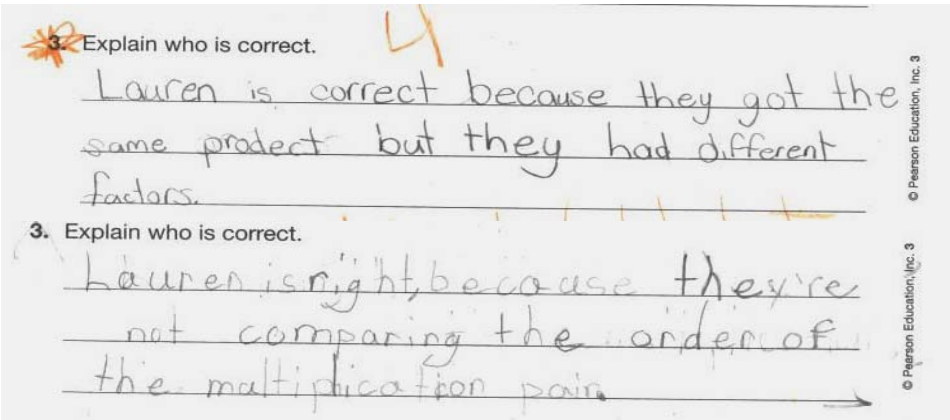
3. Explain who is correct.

Lauren is correct because they got the same prodect but they had different factors.

3. Explain who is correct.

Lauren is right, because they're not comparing the order of the multiplication pair.

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Figure 9. Student Samples – Math Communication in Action. Three students use the correct math terminology and/or examples in explaining their reasoning in solving a math class assignment problem.

Freire (1970) stated "Knowledge emerges only through invention and re-invention, through the restless, impatient, continuing, hopeful inquiry human beings pursue in the world, with the world, and with each other" (p.72). As I continue to observe my students communicate their mathematical thinking process as they solve a range of mathematical problems, I can formulate a direct connection between the student's actions and Freire's assertion on the creation of knowledge.

METHODS OF ANALYSIS

Ely, et al. (1997) emphasize the importance of ongoing analysis in an action research study. They state “One of the most fascinating – and sometimes frightening – aspects of qualitative research is its emergent nature” (p.175). In order to maximize the positive attributes of an action research study it is important for the researcher to act accordingly to what the data reveals throughout the course of the study.

Through continuous analysis of my field log contents, I was able to examine, evaluate, and question my data in gaining a comprehensive understanding of my students’ progress in math communication. As I reread my observations in the field log, I followed Bogdan and Biklen’s (1998) advice and labeled my observer comments with “OC.” By separating the comments from my actual observation it allowed me to ensure that that I was not biased in my analysis. In the margins I coded my data. Ely, et al (1997) describe coding when researchers “read and reread a portion of data and provide labels – usually notes in the margin – that identify a meaning unit” (pg. 162). I started coding the data a few weeks after my data collection commenced. In assigning codes I selected terms that encapsulated the information that I was reading. For example, when I read information on a positive student reaction to events that were occurring, I coded it “student enthusiasm.” I also coded the entries in reference to the specific math communication levels, scaffolding instructional methods, and math

communication strategies that were explicated within the observations. Towards the end of the data gathering time period, I started to organize my codes in relation to each other. In alignment to Ely, et al (1997), my next step was to classify the individual codes into bins of similar codes and arrange the bins in a graphic organizer. As I analyzed the bins within the graphic organizer, I was able to identify correlating themes that explicated my data findings. Figure 10 displays the associations that I found with my codes and bins. Figure 11 contains the theme statements that I arrived at in analyzing my bins.

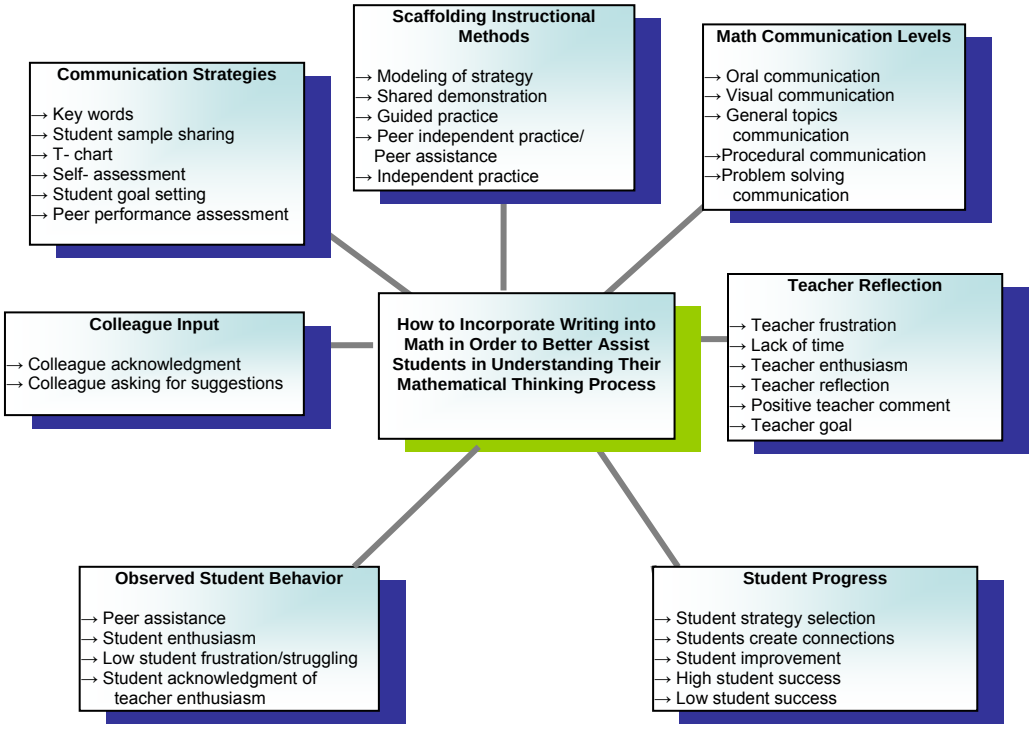

Figure 10. Codes and Bins. Illustrates the relationship among individual codes as they were assigned to bins.

1. Scaffolding strategies are an important tool in guiding math communication instruction at each student's instructional level.
2. In communicating their mathematical thinking, students progress through different levels of math communication starting with oral communication of general math topics to more inferential communication in the later stages.
3. When students are exposed to a variety of strategies that they can utilize to effectively communicate their mathematical thinking process they become more independent thinker as they employ the strategies accordingly.
4. Teacher reflection, of both positive and negative components within math communication, is essential in planning future lessons in math communication.
5. Student progress needs to be continuously assessed in order to construct lessons that employ the appropriate scaffolding techniques to meet each student's individual needs as he/she learns to use the various strategies to communicate his/her mathematical thinking process.
6. Observed student behaviors are imperative in modifying instruction in order to best assist student progress and understanding of math communication strategies.
7. Colleague input is a valuable resource in validating student progress in math communication entries in connection with grade level standards.

Figure 11. Theme Statements. Explicate how the individual codes and bins portray the gathered data.

I discovered that I was most successful in coding when the entry was no longer in my recent memory; therefore, I usually coded the data at least two weeks after the recording. When I reread observations and reflection soon after I had written them, I wasn't able to recognize components that could potentially be valuable later in the process; therefore, I tried to code my field notes at least two weeks after I wrote them.

I monitored student progress through my analysis of student work. In addition to analyzing student work, my field notes on students' conferences provided me with a continuous assessment of the students' ability to orally communicate their math thinking process as well as their ability to transfer the oral to written format.

As I read the works of Delpit (2002), Dewey (1938), Freire (1970), and Vygotsky (1978), I examined my data through each of the theorist's point of view. By writing analytical memos focused on each theorist's perspective, I analyzed my data through linguist, traditional, progressive, socioeconomic, dialogical, constructivist, and traditional "lenses." As a result, I was able to support my observations and finding using these educational philosophies.

FINDINGS

Vygotsky's (1978) statement "A well known and empirically established fact is that learning should be matched in some manner with the child's developmental level" (p. 85) encapsulates the findings of my study. I discovered math communication needs to be approached in this manner to best benefit each student. Writing provides a medium through which students are able to explicate their mathematical thinking at their own personal developmental level.

In addition Dewey (1938) describes learning as a continuum of experiences that an individual progresses through every minute of each day. Because every exposure within a person's lifetime will impact future events, it is important that educators provide a continuum of experiences that build upon each other. The learning opportunities that educators provide should assist the students in understanding how one outcome leads to the next. In my study, scaffolding strategies allowed the students to gradually become more independent at completing math communication tasks. Routman (2003) explains how scaffolding instruction can assist students in becoming independent proficient readers. Routman describe her scaffolding technique to include four phases: demonstration, shared demonstration, guided practice, and independence practice. I utilized Routman's scaffolding framework as a resource in planning my instruction of specific communication strategies. The first phase included modeling the specific strategy for the students. In the second step, shared

demonstration, the students and I would complete a sample using the specific strategy. After the initial two phases the focus shifts from the teacher to the students in guided practice and independent practice. The students then complete a task with my support. The final goal is for the students to complete a communication task independently. However, I discovered that the scaffolding framework, itself, does not bring success. It is only when the instruction is scaffolded to match the students' developmental levels does it assist the students in achieving success as well as bring forth student active participation. Many times when I first presented a strategy I combined the first two scaffolding instructional methods in order to reach most of the students, especially when students had a working knowledge of the math concept that was covered in the communication task. For example, as I modeled the strategy itself, I asked the students to participate in providing the explanations based on what they already have learned about the specific topic that I was explaining. This enabled the students who understand the topic to share their knowledge while at the same time providing a model of the strategy for the students who had trouble with the concept. In addition because the strategies build upon each other, the students were able to share and apply their expertise on the previously learned communication strategies. Individualized instruction was crucial during the integration of guided practice and independent practice of any specific communication strategy. When the students started to work on their

communication assignment, I needed to know which students could start independently, which ones would need my support from the start, which ones simply needed me to conference at the end, and which ones would need me to clarify any arising questions as they complete their entry. Even though I found scaffolding instruction to be important, I believe it is more imperative that the teacher is flexible enough to accommodate the fact that each student moves through the scaffolds at different rates. If I had moved through sequentially each time for each student, I know that high-achieving students would have been bored and unmotivated. However by integrating demonstration and shared demonstration, these students were actively involved from the start while the other students benefited from observing the strategy in action. By adapting Routman's scaffolding instruction to meet my student's individual needs, I was able to actively involve the students in activities that would help them complete a math communication task more independently in each subsequent day.

A crucial component that I discovered in using writing to assist students develop their mathematical thinking skills is that assessment of student progress needs to be continuous. Through comprehensive assessment of student progress, I was able to construct lessons that employed the appropriate scaffolding techniques to meet each student's individual needs as he/she learns to use the various strategies to communicate his/her mathematical thinking process. Assessment, however, does not need to be equated with the assignment of a

specific grade on a task. I discovered that through conferencing with individual students as they work on their communication task I was able to assess the students' mathematical thinking process in action. Through my dialogue with the students, I was able to actively determine the student's ability level in communicating his/her mathematical thinking. Freire (1970) states " . . . dialogue, which requires critical thinking, is also capable of generating critical thinking" (p. 92). By assessing student progress through conferencing with the students, I was able to not only assess a student's progress with the task, but I was also able to assist the student in improving to the next level. Too many times teachers are pressed for time, and I am no different. In fact, researchers share that a common issue within math communication is teachers' concerns over the additional amounts of time they need to incorporate into their schedules to provide feedback for the students (Baxter, Woodward, & Olson, 2005; Burns & Silbey, 2001; McIntosh & Draper, 2001; Ryan & Rillero, 1996). As I planned my study I had been optimistic that I would be able to keep up with the feedback if I followed my rotational schedule; however, once the school year started I too faced the time issue. Therefore, as my study began to evolve, and I discovered the benefit of assessing and guiding students simultaneously, I quickly decided this was the route to take. After all, once the students leave for the day and I sit down to grade students' assignments, no learning is taking place on the part of the student, and my opportunity to I assist them in being more successful in the current task has

slipped away. Regardless of the math communication task that the students were asked to perform, I had to maximize my opportunity to guide each student to reach his/her potential. I needed to combine my on going assessments of each student as well as improve the student's understanding of the concept.

As students learn to communicate their mathematical thinking process, they needed the opportunity to progress through the different levels of math communication starting with oral communication of general math topics to more inferential communication in the later stages. The importance of fostering an environment in which the students are able to naturally advance through the different stages is the major component that I was missing in my math communication instruction prior to my study. With the growing initiatives to improve students' abilities to communicate their mathematical thinking in the last few years, teachers are often asked to integrate math communication into their current math curriculum (Quinn, 1997). As I asked my students to complete communication tasks on mathematical concepts that required higher order thinking skills in the past, I wondered why so many struggled from the onset. Even when the students were able to quickly solve the problem, they still could not communicate how exactly they arrived at their answer. Although with practice students started to become more successful, it was a long, stressful, frustrating progress for many students as well as me. My study provided me with a first hand medium in discovering that like reading, spelling, math computation; math

communication needs to move along a continuum. We can not expect students to have the skills to explain how they solved inferential word problem until they have the opportunity to solve simple procedural computation problems. Similarly, we can not expect them to naturally be able to write their thinking skills until they can successfully verbalize those explanations. The students progressed from oral and visual to written communication. Pugalee (2004) indicates the importance of progressing to written communication once the students are able to orally explaining their thinking process. Pugalee states that although both oral and written communication aid students in understanding their thinking process, those that advanced to written expression were more likely to become independent problem solvers. In addition, the communication tasks needed to move along a continuum of complexity. Students started out with general topics; such as identifying the uses of math in everyday life. Then the task progressed to procedural communication; for example, explaining how they added 345+589. Once the students were successful in communication procedural topics, they were then more successful in their journey of sharing their mathematical thinking skills in complex tasks; such as words problems. Baxter, Woodward, and Olson (2005) discovered similar success when the teacher in their study presented math communication through a continuum. In their study the communication tasks ranged in four levels from recording to relating. An important consideration is that the continuum for one group of students can differ to another group. For

example, it was important for me to establish a learning environment in which the students would feel comfortable orally communicating. In their study Baxter, Woodward, and Olson were researching math communication within a class of low-achieving middle school students who often choose not to participate in class, and therefore, their first stage was for the students to record the information that the teacher shared. However, simultaneously the teacher presented a journal means through which she communicated individually with students and thus giving them an opportunity to express their feeling with the teacher. I noticed when students are able to progress naturally from general topics to procedural tasks and then to inferential problem solving, they gathered the skills they need in order to successfully meet the requirements in the next task.

After being exposed to a variety of strategies, the students had the tools necessary to explain their mathematical thinking process. Just like I do not expect the students to instinctively know how to multiply, write in paragraph form, identify the parts of the water cycle, I learned that I can not simply assign math communication tasks without teaching strategies for the students to use in explaining their thinking process. Simply because they can solve a problem does not mean they can explain how they arrived at that answer! Therefore, when working on math communication I wanted the students to have experiences using a variety of strategies so that they could select one or a combination of them in showing their mathematical thinking through writing. The key strategies that I

focused on during my study were key words, student sample sharing, t-charts, self-assessment with goal setting, and peer performance assessments.

One of the initial strategies that I introduced was the use of key words. The students learned how using explicit math terminology assists their reader in knowing exactly what they are trying to convey in our explanations. The math word wall provided the students with a visual quick reference of mathematical terms that they found beneficial.

Student samples were a valuable tool in showing students how their peers responded to particular tasks. Students realized how each student can approach each communication assignment differently, but still arrive at the correct answer. This realization brought comfort to the students in knowing that there is not only one way to solve a problem and/or explain one's mathematical thinking process. One additional benefit of using student samples was that it motivated students such as, Rich, to do their best because they wanted me to select their entry for the next sample. In addition, it provided students who were confused on a particular task multiple examples to reference.

T-charts allowed the students to organize their explanations. As the students learned more complex math concepts, their explanations of how they solved particular tasks were becoming more comprehensive. In order for the student to successfully convey the multiple steps it was important for them to learn an organizational strategy at this point. The t-chart allows the students to

simultaneously perform the needed work and explain their thinking process by having these two distinct items separated within the t-chart.

As Risk (1988) indicates the ultimate goal is for the students to be able “to set goals and identify strategies for themselves” (p. 36). The effects of self-assessment and goal setting by the students assisted the students in becoming more independent math thinkers. As students reflected on their responses based on our class conferencing of student samples and then identified the positive items that they had done as well as the things they needed to work on for the next time, they started taking a more active role in improving their math communication. It was amazing to see the students look at previous entries in deciding what they needed to include in their current entry. An important finding was that the students preferred to include an item in the entry right away rather than admitting it as an item they needed to work on next time. It become evident they did not like to admit they forgot something. They rather simply add it to their explanation right away. Either way I could assess that students knew what they needed to have in order to convey their mathematical thinking process in a clear and concise manner. Perhaps, developmentally it is easier for them to incorporate the item they forgot rather than admit they need to work on something for next time. In addition, I discovered that the students were more successful in improving their communication entries by using their self-evaluations than my feedback. The personal reflection brought the students a deeper understanding of what a

comprehensive explanation of ones' mathematical thinking process needs more effectually than any suggestions made by peers or myself.

Peer performance assessment was the strategy that the majority of the students showed a particular interest in. By having the opportunity to read one's explanation and observe a partner complete only the steps that are described not only excited students but assisted the students in realizing the importance of explaining each of the steps in an explicit manner. Peer-performance assessment was the strategy that brought the most enthusiasm and positive feedback from the students. Consequently, peer-performance assessment was also one of the strategies that brought more students to achieve success, especially struggling students such as Rachel and Jacky through peer interaction.

Vygotsky (1978) publicized ". . . learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (p. 90). As my study progressed one important observation that I encountered was how well the students naturally started discussing their writings with each other. While at first, I questioned whether or not to allow the students to interact as they complete their communication tasks because I feared that they would not keep working on the task at hand, I soon discovered the benefit of the peer interacting as I meet with the students to review their current task. As I approached different pods in coming to conference with the students, I would hear students asking each other the

prompting questions that I would ask them in our conferences. Peer interaction became a key component is providing a means for the students to develop their thinking without my direct involvement. The evaluation of student samples on the overhead through a whole class discussion format was an addition indication of how much students learn through peer dialogue.

My personal teacher reflection of both positive and negative components was an essential component in assisting students to achieve success in expressing their mathematical thinking skills. Throughout the course of my study, I needed to reflect on student work, student progress, math concepts, and presentation of strategies on an ongoing basis in order to develop lessons that best benefited my students' needs. Even though I had planned out a timeline for my study, which included a schedule for the presentation of strategies, it merely served as a framework. I could only finalize math communication levels after the previous lesson had concluded and I could assess my students' progress. For example, when I first had the students complete a math communication task on identifying the strategies they have learned about problem solving, I discovered that most students were not yet ready to write out all their steps. On that day I then focused the setting to a whole class discussion and sharing of problem solving prior to having the students write down their responses. Consequently, I then focused our next math communication lesson on a visual representation of problem solving strategies.

As I reflect on the course of my study, I find myself making a connection with previous teacher researchers in which we all faced the time constrains of scheduling and time commitment needed to make math communication beneficial to the students; however, in the end, student success seems to make it all worth while. Assemblies, snow days, math grouping, and holidays all presented factors beyond my control that influenced the math communication time period. Even though I observed that consistency definitely plays a significant part in student success, I also noticed that students are very resilient and are able to adapt as factors arise. My students achieved higher success as well as more consistent improvement when I was able to devote two twenty-minute periods to math communication. One the first day I presented the strategy through demonstration and/or shared demonstrating and the students then had an opportunity to work on a task. On the second day, we could conference on student samples, and then the students self-evaluated themselves and made any needed revisions. However, with careful attention to schedules I could have students work on communication assignments that focused on math concepts that did not need as much concentration on weeks that I could not devote two periods to math communication.

As I share and reflect on my finding, I believe it is crucial to share that the focus of my study was the students not math communication. With my focal point being centralized on the students, I recognized the important of adapting, altering,

and modifying instructional methods and strategies to meet each student's individual needs. The smiles on the students' faces when they realize they have achieved success or when they realized that I was proud of what they completed confirmed that I was focused on the vital ingredient in education – the students!! In addition it was amazing to see how much the students acknowledged my love for math. Surveys, informal interviews, as well as student conversations (both in homeroom as well as in math classes) all indicated how much the students knew my excitement for math. Even my grade level colleagues often comment on my passion for math, and how my homeroom students display my enthusiasm in their math communication tasks during math class. Dewey (1938) best described it when he stated, "The most important attitude that can be formed is that of desire to go on learning" (p.48). My study demonstrated that the attitude starts with the teacher!!! "When they recognize that we believe in them, then they come to trust us, accept us, to identify with us and to emulate us" (Delpit, 2002, p.46).

WHERE DO WE GO FROM HERE

As I reflect on the student population of my study, I acknowledge the fact that the lack of diversity presented a limitation on my findings. I am left wondering what supplementary scaffolding methods and strategies I would have needed to employ in working with students with an I.E.P. and/or E.S.L. students. As I continue in my teaching career, I will have the opportunity to encounter such students in my class again. I look forward to working with these students in continuing my research in identifying the best possible way to accurately time scaffolding strategies to assist them in reaching their potential in explaining their mathematical thinking process. As I reflect on my research, I perceive picture clues to be a possible visual strategy to incorporate in assisting the students. In addition, I question if having a personal folder with the word wall terminology posters would assist the students who have visual, processing, and/or focusing problems that may impact them in reference the math word wall.

An additional topic that I will continue to research is what other communication strategies I can use in assisting my students communicate their mathematical thinking process. Key words, sharing student samples, t-charts, self-assessment, goal setting, and peer performance assessment were all valuable tools; however, in order to teach each student within his/her zone of proximal development (Vygotsky, 1978), I acknowledge that these strategies are not the cure all. Each student possesses distinct learning characteristics, and a teacher

needs to have a comprehensive bag of strategies to work from in assisting each student succeed to his/her potential.

A further research topic would focus on time management in incorporating math communication within math instruction. Time is a never-ending factor that negatively impacts many researchers who study incorporating writing in math communication. There doesn’t seem to be enough time in the school day, and with the additional demands that are impacting education; it is not surprising that time is a constant factor. Where in the academic schedule would math communication best benefit the students? A question I will continue to ponder and investigate in my career.

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APPENDIXES

Appendix A: Parent Consent Form

September 2005

Dear Parents,

During the 2005-2006 school year, I will be working towards earning my Master's degree in Curriculum and Instruction at Moravian College. The Master's program at Moravian College is designed to provide teachers with the opportunity and guidance to study their own teaching methods through both literature as well as action research: as a result, the research guides teachers in designing lessons that will provide students with the best possible learning experiences.

My thesis research will focus on how to integrate writing into math instruction in order to encourage and assist students in understanding their mathematical thinking process and be more successful in math assessments. I feel that the ability to use writing to explain one's mathematical processes can be very beneficial to students in becoming proficient problem solvers across a variety of situations and including writing in math instruction will assist the students in this process. As part of my study, I will be conducting student observations and collecting samples of student work. I will be conducting surveys and interviews with the students on their views of mathematics and writing within mathematics instruction.

Even though all of the students will be involved in the integration of the various writing techniques within our math curriculum throughout the year, the participation in the research study is entirely voluntary and will not affect your child's grade in any way. He/she may also withdraw from the study at any point. If he/she withdraws or decides not to take a part in the study, I will not use any information (observations, work samples, surveys, and/or interviews) pertaining to your child in my study.

All of the students' names will be kept confidential. In fact, for the purpose of my study each student, faculty members, and our school will be assigned a pseudonym, a fictitious name. I will then use the pseudonyms in any of my observations, notes, as well as all written results of my findings of my thesis. In order to preserve each student's confidentially, I may also need to make minor changes in some student samples. Any changes performed to ensure student confidentially will be completed in a manner in which it will not alter the essence and views that the student was displaying in his/her data sample.

After reading this letter if you have any questions or concerns about my study, please feel free to contact me at school (phone number) or e-mail me at (email address). If you do not need any further information, please sign the bottom portion of this letter and return it to school with your child. You may keep the duplicate copy of the letter for future reference.

You may also contact my professor, Dr. Joseph Shosh, at Moravian College. He can be reached by phone at (phone number) or email at jshosh@moravian.edu.

Sincerely,

Mrs. Hillard

I attest that I am the student's parent or legal guardian, and that I read and understand this letter and give consent for my child's data to be used in Mrs. Hillard's study. I have also received a copy of the letter for future reference.

Student's name: _____ Date: _____

Parent/Legal guardian's signature _____

Appendix B: Principal Consent Form

August 29, 2005

Dear (principal's name),

During the 2005-2006, I will be working towards earning my Master's degree in Curriculum and Instruction at Moravian College. The Master's program at Moravian College is designed to provide teachers with the opportunity and guidance to study their own teaching methods through both literature as well as action research; and as a result, the research guides teachers in designing lessons that will provide students with the best possible learning experiences.

My thesis research focuses on how to integrate writing into math instruction in order to encourage and assist students in understanding their mathematical thinking process and be more successful in math assessments. I feel that the ability to use writing to explain one's mathematical processes can be very beneficial to students in becoming proficient problem solvers across a variety of situations and including writing in math instruction will assist the students in this process. As part of my study, I will be conducting student observations and collecting samples of student work. I will also be conducting surveys and interviews with the students on their views of mathematics and writing within mathematics instruction.

Even though all of the students will be involved in the integration of the various writing techniques within our curriculum topics for this semester, the participation in the research study is entirely voluntary and will not affect any child's grade in any way. Each child may also withdraw from the study at any point. If he/she withdraws or decides not to take a part in the study, I will not use any information (observations, work samples, surveys, and/or interviews) pertaining to that child in my study.

All of the students' names will be kept confidential. In fact, for the purpose of my study each student, faculty members, and our school will be assigned a pseudonym. I will then use the pseudonyms in any of my observations, notes, as well as all written results of my findings. In order to preserve each student's confidentially, I may also need to make minor changes in some student samples. Any changes performed to ensure student confidentially will be completed in a manner in which it will not alter the essence and views which the student was displaying in his/her data sample.

After reading this letter if you have any questions or concerns about my study, please feel free to contact me personally, by phone (phone number), or e-mail me at (email address). If you do not need any further information, please sign the bottom portion of this letter indicating your consent for my study. You may keep the duplicate copy of the letter for future reference.

You may also contact my professor, Dr. Joseph Shosh, at Moravian College. He can be reached by phone at (phone number) or email at (email address).

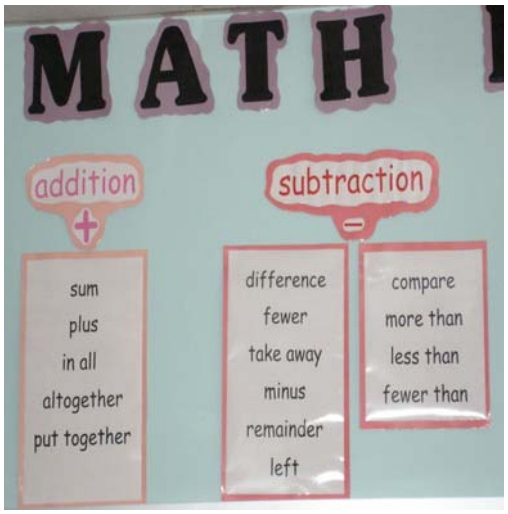
Sincerely,

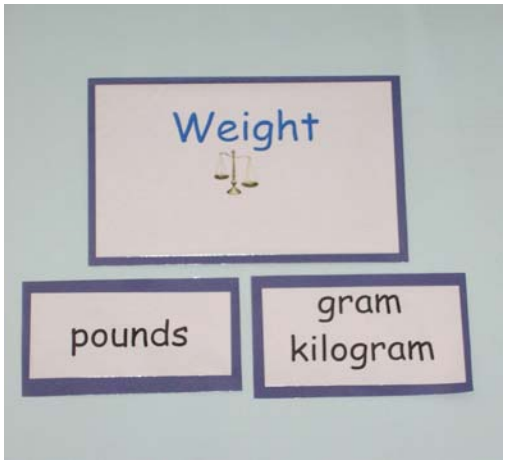
Mrs. Hillard

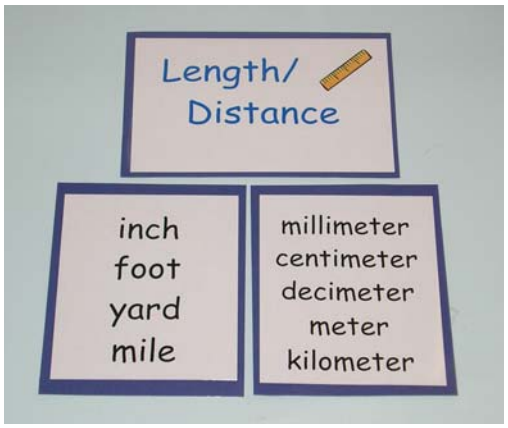
I attest that I am the principal of the teacher conducting this research study, and that I read and understand this consent form. Mrs. Hillard has my permission to conduct this study at (school name). I have also received a copy of this letter for future reference.

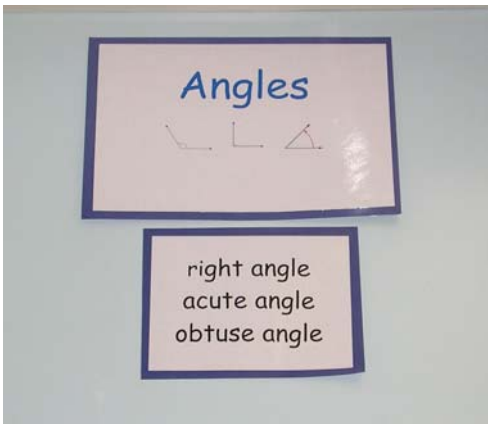
Principal's signature _____ Date: _____

Appendix C: Math Communication Posters










Place Value
ones
tens
hundreds
thousands
ten thousands
hundred thousands
millions


Solid Figures
cube
sphere
pyramid
cone
cylinder
rectangular prism

Appendix D: Initial Student Survey

Math Insights

Circle a response for each statement.
Then explain your answer as best as you can.

1. I enjoy math class. Never Sometimes Most of the time Always

   Explain.

2. I am a good problem solver. Never Sometimes Most of the time Always

   Explain.

3. I wrote in math class last year. Never Sometimes Most of the time Always

   Explain.

4. I think math homework is easy. Never Sometimes Most of the time Always

   Explain.

Finish each sentence with as many details as you can.

5. When I think of the word math, I think of

6. My favorite part of math is

7. My least favorite part of math is

Please write your name on the line on the back of this survey.

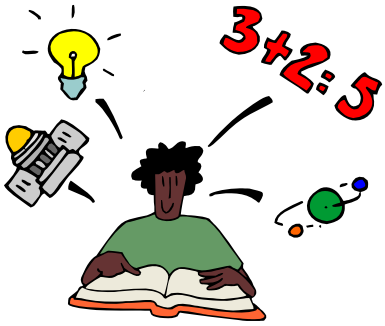
Name _____

Appendix E: General Math Communication Template - Why We Need Math

Name _____

When do you use math in your daily life?

Think of the examples in the book A Day with No Math to help you get started.



_____

_____

_____

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_____

_____

_____

_____

_____

Appendix F: Sample Procedural Math Communication Templates

Name _____

Math Communication

Let's try it together:

Place the following numbers in order from least to greatest. Explain each of your steps.

784 529 1,536

_____

_____

_____

_____

_____

_____

Your turn:

Place the following numbers in order from least to greatest. Explain each of your steps.

352 635 28,896

_____

_____

_____

_____

_____

_____

Name _____

Look over your chapter 1 test.

Pick a problem and explain the steps you took to solve it as if you were showing a second grader how to do the problem.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Name _____

Round 536 to the nearest 10.

Explain each of your steps.

Name _____

Math Communication

If you had to explain to someone how to tell time, what would you say? Use minute hand and hour hand to explain 5:28 p.m.

- **Show** all your work.
- **Write why** you did each step.

Appendix G: Math Communication Visual Problem Solving Template

Name _____

These are the steps I use

I read the problem.
I double check my work. (I check to make sure I added or subtracted correctly, or that my picture, graph, or table shows the right information from the problem.
I make sure my answer makes sense and answers the question correctly.
I check to make sure I understand the question.
I write the answer in a complete sentence.
I use a strategy to solve the problem.
I find the important information in the problem.

Appendix H: Mid-Study Student Survey

Math Insights

Circle a response for each statement.
Then explain your answer as best as you can.

1. I enjoy math communication. Never Sometimes Most of the time Always

   Explain.

2. I am a good problem solver. Never Sometimes Most of the time Always

   Explain.

3. I understand my math homework. Never Sometimes Most of the time Always

   Explain.

Finish each sentence with as many details as you can.

4. When I think of math communication, I think of

5. My favorite part of math communication is

6. My least favorite part of math communication is

7. Why do you think we spend time on math communication?

Please write your name on the line on the back of this survey.

Name _____