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**MATHEMATICAL PROBLEM SOLVING USING  
DIALOGUE IN A THIRD GRADE CLASSROOM**

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## **Abstract**

This qualitative research study investigated the observed behaviors and reported experiences of 3<sup>rd</sup> grade students when implementing dialogue in a mathematical problem solving class. Eighteen students participated in the study. Problem solving activities involving logical and reasoning skills, higher level thinking, multiple step word problems, Sudoku puzzles, and number equations involving number sense and choosing correct operations were completed as a means of producing dialogue and discussion in small collaborative groups. Data were gathered using observation of dialogue between the students and student work was collected and analyzed. A pre-survey and post-survey measured student attitudes and perceptions and also one-on-one student interviews were conducted to gather more specific information. This study suggested that when students are organized in collaborative groups and given the opportunity to discuss mathematical problem solving their depth of mathematical understanding improves. The study also suggests that when students are given autonomy to choose their assignments it solidified their mathematical vocabulary and increased their motivation and self-confidence.

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### **Researcher's Stance**

My journey began with a love for children and the excitement and energy I felt when I would interact with them in a variety of environments. I had begun my college career looking for a job in which I would help people, not necessarily children. I registered for classes involving anatomy, biology, etc., and found that something was missing. All of these courses would lead to helping people rehabilitate after experiencing traumatic events, but did not possess the excitement and energy that children emit at any given moment. I enjoyed working with children through various volunteer opportunities and felt the vibe throughout the school day in classrooms.

I was hoping I could land a student teaching placement in the Lehigh Valley because the chances of being hired afterwards would have been greater. But instead, I was placed into the School District of Philadelphia to complete my student teaching. I had completed pre-service experiences in some schools in the district, but had some anxiety about how I would succeed. Professors constantly would tell us that knowing your students is the single most valuable information teachers need when working with your students. A way to bridge the cultural gaps I had with my students would be through dialogue and conversation. We were different culturally, but we all spoke English. Even though there were linguistic and vocabulary differences, verbal communication was the way I would learn from my students. I would constantly think about how different the students I

have in front of me are compared to me. I thought it was almost as if they were from a different world.

Knowing nothing worthwhile in life comes easy I woke up with tons of energy and excitement ready to tackle my experience head on. My cooperating teacher, with whom I still keep in contact 9 years later, was positive, insightful, and a perfect mentor for my personality. I spent the most rewarding semester with third grade students with whom I thought I had no similarities, but finally discovered how to bridge our worlds so we could learn from one another.

The opportunity arose to teach on the other side of the world when I chose to broaden my experiences and student teach in London, England. These students were very diverse, some speaking three different languages, and all coming from financially strapped families. How could I use verbal dialogue to learn more about these students? Many of my students spoke English as a second language, but all of my students had a different dialect from mine and also used very different vocabulary. Moving to a large city for the next six weeks away from the comfort and conveniences of home quickly proved to be more of a challenge than ever anticipated. A national curriculum, students with autism that are fully included, new vocabulary, cultures, languages, different styles of handwriting, and living in a flat with cockroaches questioned my endurance and drive to teach in this new environment. Each day was a struggle to maintain control in an environment where students could act out emotionally and physically at any moment. I

continued to do what my professors stressed, try to connect with my students and find common ground even when I felt we had no connection. After a few weeks we were able to fully understand one another and eventually we achieved success. I was able to bond with several, but not all of my students. Unfortunately, it occurred at the end of my placement and I had graduation to look forward to.

Over 200 applications later, day-by-day substituting was all I was offered. I chose two local districts hoping that I had the ability to work everyday and possibly teach summer or after school programs. After a few months of never turning down a school, I was offered a fifth grade short-term substitute position in an alternative school. Smaller class sizes, teacher assistants, supportive staff, family, medical, and counseling services provided for our families, and laptop initiatives for each of our students appeared to provide our students with the most support needed to be successful. I was hesitant to accept the position because I had heard from the staff that they had difficulty looking for a teacher willing to accept the position because of the behavioral challenges that these students exhibited. After only a short time to make the decision, I accepted the position and was excited to finally call a classroom my home, even if it was only for a few months.

I would listen to other faculty members talk about the heartbreaking situations many of the students faced in their short lives. Each day was a challenge, but my instinct told me to develop a relationship with my students and

focus on my strong leaders. They made themselves known immediately during my lessons. These students observed my every move, just as I was carefully observing them. Keeping expectations consistent and my emotions under control in front of the students, I would vent my frustrations to other staff members who worked with the same students. This would be the only way I could deal with the raw emotions I was feeling regarding the teaching of these challenging students. I questioned the career choice I made of becoming a teacher.

With the following school year quickly approaching, I was hoping to interview for a permanent position. One morning, I received a phone call from the district that I substituted in to teach AM and PM kindergarten for an extended period of time. I was ecstatic! I was able to decorate the classroom and move in with materials that I had purchased to use with my students. What a drastic change from what I had the previous year, teaching kindergarteners sounds, letters, and how to write their first names. As the months progressed I, as well as the students learned from one another. Different challenges arose such as how to help students deal with separation anxiety, feel comfortable enough to ask questions and socialize with their classmates, and manage a room full of energetic five year olds.

Again, I was able to push through the challenges and continue to improve my instruction and management to become a better teacher. Teaching these students how to socialize and get along through conversation and not other means

was at times difficult. The cultural and linguistic diversity of my students forced me to teach them in an individual rather than whole group setting. The years progressed, and I continued to develop my skills, but didn't have a full time position. I continued to work in the same urban elementary school, teaching after school programs as well. Finally, the summer of 2006 arrived and I was teaching summer school in a building without air conditioning on the second floor. The principal walked up to me and offered me a third grade position in the building where I felt most comfortable. I nearly went into shock, but quickly accepted.

Five years later, I remained teaching third grade students in the same classroom. Each year was different in so many ways compared to the last. I always had a diverse group of students, where the majority were impoverished, spoke different languages, predominately Spanish, and came from single parent households. But, I worked hard each year to improve student retention, move my students at least a year in reading, math, and writing, increase motivation, make school enjoyable for my students, and give families and students the things they needed to be successful.

Yet another hurdle was being able to teach children who didn't speak English as their first language or who had limited English proficiency because they came from homes where communication was hardly used. I found that the only way to communicate with many parents was to get in contact with another staff member who spoke Spanish and relay the information. Ruby Payne found

that discourse plays a vital role in success in American schooling. “The use of formal register is further complicated by the fact that these students do not have the vocabulary or the knowledge of sentence structure and syntax to use formal register. When student conversations in the casual register are observed, much of the meaning comes not from the word choices, but from the non-verbal assists” (Payne, 2005).

Not until I worked with these students and families for a few years did the challenges these children face really set in. A night would not go by without my taking their stories home and crying about the traumatic events some of my students have been through. Ninety percent of my students receive free or reduced lunch and have faced such difficulties as homelessness, lead poisoning, drug and/or alcoholic addictions, and incarceration within the immediate family. Almost one-third of my students do not speak English as their first language, and nearly 85% of their parents never graduated from high school. Taking these challenges into account I wanted to create an environment in which my students felt safe and comfortable.

I remember asking one of my students a few years ago what his family spoke about at the table at night while eating dinner. His response was, “Miss, I’m not allowed to talk when we’re eating, my dad just tells me to shut up and eat.” The response was shocking to me; I recalled how every night at my dinner table while growing up we had true conversations about what happened throughout the

day at school. As I contemplated that student's statement, I wondered what role dialogue and discussion in the home and at school played in the educational process. Children brought up in home environments where they are not allowed to speak or are just told what to do, miss out on conversation and having true dialogue.

Expanding vocabulary and correct reinforcement of the English language are rarely practiced in these homes. As noted by Gallagher (2009, p. 32) Wolf states, "By kindergarten, a gap of 32 million words already separates children in linguistically impoverished homes from their more stimulated peers." I wanted to create an environment in which students can feel their opinion matters and that what they have to say is important.

High poverty schools struggle to achieve the same levels of proficiency in all academic areas that middle and upper class schools already achieve in high stakes testing. When we look at data that our school receives from the PSSAs, our students normally perform lower in problem solving and open ended questions compared to other areas of academic learning.

The pressures that districts are under to have students perform well on standardized tests force educators and administrators to focus on basic skills rather than on higher level thinking. I personally succumbed to the pressure; I wanted to ensure that my students would be successful on the majority of the tests so I would put instruction focusing on open-ended responses last.

Throughout my years of teaching diverse populations, communication and dialogue have been the most vital components to successful instruction. Therefore developing my pilot study around dialogue was easily decided upon. We, as teachers, feel that our kids can succeed when basic computation strategy questions are given to our students on standardized tests, but when these questions are embedded within word problems or given in other contexts, our students struggle. My research question came from these discussions with colleagues and also the reading of Using Discourse Analysis to Improve Classroom Interaction (Rex & Schiller, 2009), which stresses the value of classroom conversation in strengthening students' understanding of content in all subject areas.

After I was beginning to feel comfortable with teaching this population of students, this summer I was moved to a building where less than 5% of the students are on free or reduced lunch and more than 90% of the student population is proficient on the PSSAs. Currently my students have not experienced any of the traumatic events that my prior students had. Every student has consistent expectations and support to complete homework assignments from well-educated parents, stable home environments, access to quality health care, and nutritious food on the table every night. They have books, resources, and access to transportation if they need assistance on learning something new. My students participate in sports, community activities, and educational programs in the summer. They also take vacations all over the world in order to broaden their

horizons to become well-rounded citizens. I had begun to question how I was going to connect these two very different student populations to my thesis question but, to my relief, one similarity still holds true among the scores in my current placement: Even though my students are proficient at a much higher level overall in reading and math, their scores for open-ended responses remain the lowest out of all the subjects tested on standardized tests.

When the focus of my research came clear, I began to wonder why my students had difficulty with explanations in mathematics. These difficulties were always in response to word problems. Possibly the reading and comprehending of what the problem was asking is part of the difficulty. As I read research articles and spoke with my colleagues, I thought that adding discussion and dialogue in problem solving would move more children along the thought process that was required when solving word problems. The powerful strategy of using pictures, numbers, and words to solve problems would also be a way to assist my students to complete better explanations. The strategy of adding dialogue and discussion to the mathematical problem solving class will hopefully improve the success of my students in the area of problem solving.

My educational stance is based on the core beliefs that a nurturing environment, academic knowledge, and learning how to learn are critical to academic success. These beliefs can be combined through cooperative learning groups in which students feel comfortable with discussion and safe from criticism.

The power to attempt higher level thinking with the formation of a solid foundation in academic knowledge and metacognitive thinking are necessary to decide how discussion can improve mathematical understanding. When working with a diverse population of students, discussion and dialogue will provide my students the opportunity to use English in an academic setting along with the powerful strategy of small heterogeneous groups to target their mathematical understanding of word problems. Therefore, my research question is: What are the observed and reported experiences of third grade students when implementing discussion and dialogue during mathematical problem solving?

## Literature Review

### Introduction

Problem solving is key to enhancing mathematical understanding for students so that they are engaging in higher-level thinking and applying learned strategies in context. “When solving story problems, for example, children need to (a) understand the language and factual information in the problem, (b) translate the problem with relevant information to create an adequate mental representation, (c) devise and monitor a solution plan, and (d) execute adequate procedural calculations” (Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor, 2007, p. 283). The importance of the skill of reading is apparent in the understanding and comprehension of the word problem itself. Even if the word problems are read aloud to the students, the working memory is involved in the processing of the language and computational skills determine the accuracy of the answer (Fuchs, Fuchs, Stuebing, Fletcher, Hamlett, and Lambert, 2008). Therefore, the teaching of problem solving is complex because it involves a multitude of content areas, not just mathematics.

One way to enhance the understanding of word problems is through dialogue in small collaborative groups. The National Council of Teachers of Mathematics supports “... interactive, discussion-based mathematics classrooms” (NCTM, 2000). Students working together and discussing their understanding of what is happening in the problem provide the foundation so that the mathematical

process can begin to arrive at a solution. Appropriately structured problem solving situations in the classroom modeled by the teacher and practiced by the students promotes mathematical understanding (Forman and Ansell, 2002).

“We propose that an essential feature of learning is that it creates the zone of proximal development; that is, learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers.” (Vygotsky, 1978, p. 90).

### **General Strategy and Schematic Based Instruction**

Over the years, two approaches to teaching problem solving in the classroom are the traditional approach called general strategy instruction (GSI) and schematic based instruction (SBI). The traditional approach incorporates problem solving activities that involve the specific skill in the lesson and the instruction may include the practice of using key words to clue students as to which strategy to use to solve the problem (Jitendra, et al., 2007). The teaching of key words as a strategy to accurately choose the mathematical operation can lead to mistakes because specific terms are misleading (Jitendra et al., 2007).

On the other hand, the teaching of problem solving through schema-based instruction addresses the mathematical structure, the semantic categories, which can lead to successful problem solving (Jitendra et al., 2007). Elementary lessons in problem solving focusing on addition and subtraction are normally organized

into four categories: change, combine, compare, and equalize (Jitendra et al., 2007). Jitendra et al. (2007) also noted that SBI targets the importance of the “part-part-whole” concept and how each piece is related to the others.

In a study conducted by Griffin and Jitendra (2009), two groups of students were given different types of instruction when solving math word problems. One group received SBI and the other received GSI. They were given pre and post tests over the course of 18 weeks and measured periodically throughout for growth. One strategy was not favored over the other for academic gains, but overall both strategies were deemed to be successful.

Each of the preceding strategies is used in classrooms across the world, but NCTM supports the addition of dialogue and discussion in mathematics because it pushes students to develop their understanding of the skills and strategies, not just the memorization of the procedural algorithms (NCTM, 2000). The terms discussion and dialogue are going to be used throughout as both meaning the exchange and transmission of ideas and thoughts to another person so that the receiving individual understands the intended meaning.

Possibly the type of instruction is not the most important reason students struggle with problem solving. Supplemental curriculum materials, such as mobile-devices, according to Lan, Sung, Tan, Lin, and Chang (2010), added motivation and a novel object (tablet personal computers) to assist the teaching of estimation in a problem-solving situation. The technology allows the students to

use virtual sticky notes to make public or private their thought process when attempting to solve word problems. The control group received actual sticky notes, whereas the experimental group had virtual sticky notes. The results from a pre and post curriculum based assessment revealed a significant difference in scores between both groups, where initially in the pre-test there was not a statistically significant difference. The students in the experimental group used the estimation strategies taught to solve real-world problems compared to the control group. They were more likely to apply what they were taught using the mobile devices than the group with no access to the technology. Instead of students participating in dialogue in a conversation setting, they were participating in discussions through the devices where each student had access to what others were thinking.

Fuchs, Seethaler, Powell, Fuchs D., Hamlett, and Fletcher (2008) used discussion and collaboration with peer tutors to teach the transfer of problem solving skills. This study is very similar to Jitendra, Griffin, Haria, Leh, Adams, and Kaduvettoor (2007) and Jitendra, Griffin, Deatline-Buchman, and Sczesniak (2007), using schematic based instruction to teach the understanding of the mathematical structure of the word problem, recognizing the schema, and solving the problem. The major difference in Fuchs' study is the addition of the skill transfer. Students with reading and/or math difficulties have a difficult time transferring the newly learned material into a different context. Direct instruction

and the use of tutors for these students were deemed. Discussion and collaboration with the peer tutors provided the necessary support for the success of these individuals.

In DeCorte, Verschaffel, and Masui's 2004 study, students were taught using the basic model of producing a mental representation, deciding how to solve the problem, devising a plan, carrying out the plan, making a solution and then checking for accuracy. This model uses content, language, intervention, and assessment to guide instruction. Instructional supports, such as whole group instruction, cooperative-learning groups using dialogue, and eventually metacognitive tasks completed independently were slowly removed when students became competent. Questionnaires were implemented on a Likert-type scale to discover student attitudes towards problem solving. The results were positive in all areas. Students favored collaboration over more traditionally constructed classes.

Discussion and cooperative groups were used in Huang (2004), along with familiar and unfamiliar contexts within the word problems. Forty-eight 4<sup>th</sup> graders were included in this study and were placed into groups based on the scores of a pre-test on multiplication understanding. The students were then paired up heterogeneously also based on the same assessment. They were given as much time as they needed to complete the assignments and were also interviewed at the conclusion of the study. Included in his results were the findings that students

actually spent more time on the familiar context problems than on the unfamiliar context problems. Based on student responses in interviews and in journals, one of the reasons the students gave was that the unfamiliar context problems had less numbers and therefore were easier to solve.

The learning environment plays a significant role in the problem solving process as well. Teachers that have created an open learning environment provide all students with the likelihood that they will feel comfortable when engaging in challenging material, such as problem solving. As noted by McCrone (2005), Manouchehri & Enderson (1999) found that “small-group and whole-class discussions encourage students to develop a more reflective stance as they take ownership of their contributions and learn to justify them in the face of questions from others.”

How students engage in mathematical dialogue and the behaviors students exhibit in a collaborative environment are also recurring themes among Esmonde (2009), Zack and Graves (2001), and Weber et al. (2008). As long as students were taught how to act in problem solving groups and the teacher had consistent expectations, effective dialogue occurred. Cazden (2001), McCrone (2005), and Morrone, Harkness, D'Ambrosio, and Caulfield (2004) noted the role of the teacher as the facilitator, no longer the educational superior. Weber used the social norms to his advantage when he conducted his research. The students were working in groups and each student had to justify to his/her group members how

they solved the problem, the group members would then evaluate the verbal responses. The justification of each of the students' responses engaged all learners in higher-level thinking (Weber et al., 2008). A consideration to note in this study is the fact that once the teacher left one group and moved to another, student discussion changed.

### **Problem Solving and Dialogue**

The National Council of Teachers of Mathematics (1991) recommends that students receive content based rather than skill based mathematics instruction so that they can solve problems in real world settings in all content areas. They also state that students experience and engage in mathematical thinking and language so that they are able to understand math, not just memorize facts. NCTM continues by saying teachers' instructional strategies must include oral questioning, problem posing and solving, practice with mathematical vocabulary where students are expected to explain verbally how they solve problems, and class discussions and dialogue (2000). The communication standards also supported by NCTM state that "Instructional programs from pre-kindergarten through grade 12 should enable all students to: organize and consolidate their mathematical thinking through communication, communicate their mathematical thinking coherently and clearly to peers, teachers, and others, analyze and

evaluate the mathematical thinking and strategies of others, and use the language of mathematics to express mathematical ideas precisely” (NCTM, 2000).

Kierean (2001) examined discourse in a mathematics classroom using six pairs of 13-year-old students learning algebraic concepts. The students were above average measured on numerous assessments compared to the remaining students in that same grade level. Individual pre-tests and post-tests were given to measure growth throughout the study. In the beginning, the students solved problems with their partners and then followed up with individual writing samples explaining their work, and then solved similar skill problems to what they completed in pairs. After conducting discourse analysis two results occurred: Not only did the discourse between partners improve the mathematical thinking between both students, the private discourse the students exhibited while solving the problems individually also contributed to their understanding. Other unexpected results occurred as well. During several instances, dialogue that occurred between the partners might have been beneficial for one student, but not the other. Also, the academically weaker students in the partnerships followed the guidance of the stronger student and assumed that their partner was correct in their explanations. The last observation of the study was that when neither of the students understood how to solve the problem the students worked independently and used self-dialogue to attempt to answer the problem.

Johnstone et al. (2006) found that discussion that occurred at the same time students were engaged in the problem solving process was more accurate than if the students were asked for explanations after they completed the activity. The fact that learned information moved from the short term memory into long-term storage explains this phenomenon. Esmonde (2009) and Weber, Maher, Powell, and Lee (2008) agree along with Johnstone et al. (2006) that discussion implemented in a problem solving math class will increase student understanding and lead students through the problem solving process.

Weber et al. (2008) conducted a study using 24 students from an urban area in New Jersey. This study was part of a longitudinal study that lasted three years. Students were placed into collaborative groups and engage themselves in dialogue. After completing the initial problems, the students were required to justify and explain their answers to the whole group. Finally, the students would then engage in a whole class discussion and decide whether their classmates' answers were correct or not and why. The researchers concluded that learning took place within the discussion because the students had to decide if they agreed or disagreed with their classmates' answers, which pushed the students to think at higher levels.

Zack and Graves (2001) conducted a similar study to Weber et al. (2008). The students were fifth graders in a private school who were comfortable with engaging in group discussions and produced written explanations in all content

areas since the beginning of the school year. Groups of four students were organized heterogeneously by the teacher and participated in joint mathematical activities. As a pre-assessment, the students were assigned an independent problem in their math logs. They then worked collaboratively to solve a similar skill problem and the teacher observed the dialogue. The researchers videotaped all sessions of collaboration in order to collect all dialogue and social interactions. Zack and Graves (2001) concluded that dialogue in mathematical problem solving assists all students in developing their understanding, but the behaviors of engaging students in verbal communication is more difficult than first thought. Each student's thoughts are heard, interpreted by other group members, and then responded to by the other students. Therefore, all dialogue builds through a process of all students' mathematical thinking. The conversation and dialogue that the students engage in is constructed from the mathematical thinking and understanding of the activities that they are participating in.

### **Collaboration/Cooperative Grouping**

One goal for mathematical discussion is that all students are engaged in discussion at higher levels all of the time, not just when the teacher is in close proximity. McCrone (2005) concluded that even though students learn throughout the year to justify and explain verbally and in written form responses to mathematics problems, they still look for teacher approval and acceptance, which is dually noted by Forman and Cazden (1985).

The role of the teacher in peer-directed groups needs consideration in the success of students in problem solving activities. Webb, Nemer, Kersting, Ing, and Forrest (2004) conducted a teacher action research study that targeted specific behaviors of students working in collaborative groups and their interactions during problem solving activities as well as how the teacher responded to students using dialogue. In this study, teachers were mostly using initiation-response-evaluation instructional discourse within the small groups when the students asked for help. In summary, the students modeled the teachers' instructional discourse patterns. Because the teacher only focused on basic understanding of the process, the students were not pushed to higher cognitive levels of understanding. An interesting note in the study was specific students would monitor other students for understanding on a deeper level even though it was not modeled by the instructor.

“...When used during problem solving situations, cooperative learning encourages students to collaborate with others rather than compete against them...puts students in situations where they learn that reading, writing, listening, and speaking in a cooperative manner...builds leadership, decision-making, and conflict management skills, and ideally, positive attitudes toward mathematics” (Muth, 1997).

### **Diversity/Dialogue & Discussion**

In order for all students to grasp the academic knowledge, vocabulary, content, and skills, the teacher must provide background knowledge or other content specific knowledge so the connection of new material to prior knowledge can be made. In addition to the knowledge learned, vocabulary words, also connected to the new knowledge, must also be acquired. Mathematical language and vocabulary instruction are intertwined and are directly connected to the understanding of mathematics and problem solving. The difficulty with solving word problems may be caused by not understanding the language the word problem is using or other vocabulary terms the students are not familiar with. If English Language Learners are organized into groups with students who speak English as their first language, dialogue and discussion can improve the understanding of the context of the problem, so the mathematical process can occur.

In a study conducted by Abedi and Lord (2001), English Language Learners were given modified word problems to solve. The complexity of specific words and phrases throughout the word problems was removed, but the content and mathematical vocabulary remained unchanged. Through the use of student interviews, and various assignments, results were that students preferred the adapted versions of the math word problems to the original versions. The adapted problems were easier to comprehend and understand, even for native English

students. Students across all ability levels were included in this study, as well as students from all socioeconomic statuses.

Overall, students from high income levels scored higher than students from low socioeconomic status, and ELL students scored lower than non-ELL students. Students from the lower ability math groups scored better on the modified version of the math assessment than the higher ability students on the same assessment. Thus, students from low SES or students who speak a language other than English perform better on adapted versions of assessments than non-ELL students. The key is to adapt the vocabulary, not the content or mathematical understanding.

Martiniello (2007) and Elbers and de Haan (2005) found that students were more successful when they were able to fully comprehend the text and apply a visual representation of their understanding for all students. This was true for all students, not just ELLs. Elbers and de Haan (2005) used structured peer groups with native and ELL students so they could assist each other in the understanding of specific vocabulary within the word problems. Another relevant discovery occurred at the end of their study: when students were too focused on the vocabulary, the mathematical aspect of the problem solving was ignored and the students often made mistakes in the solutions.

The same strategy of removing the linguistic complexity used by both Martiniello (2007) and Abedi and Lord (2001) was also used by Bernardo (1999).

In the Bernardo study, students with higher abilities in mathematical understanding and processes showed higher successes when the assessments were given in their first language and the language complexity was removed. Instructional practices that ensure student comprehension of the word problems and provide the direct relationship between what is known and the unknown variable in problem solving situations will improve the outcome in assessments of these students (Bernardo, 1999).

The majority of the research conducted focused on young, elementary age students that learn using concrete thought, but the move to abstract thinking and higher-level mathematics is also a concern. Bernardo (2005) continued his research with bilingual students and questioned whether when moving to more abstract thinking in problem solving situations, the teachers and students could use the same strategies to complete the word problems effectively. This study contradicted his prior research where language was a primary factor in student understanding. Whether the problem is in their first or second language is not a factor for older students who have a solid understanding of the structure and process the word problem provides. As noted previously by Martiniello (2007) and Abedi and Lord (2001), as long as the instructional practices include visual representations and also provide schema and background knowledge for the word problems, ELLs and non-ELLs will have success in problem solving activities.

Another component to mathematical understanding supported by the National Council of Teachers of Mathematics (1991) is the application of problem solving situations to the real world. NCTM states that students will vest more into the learning of new material if they see how they will use it in their everyday life. Bernardo and Calleja (2005) conducted a study where they focused on real world problem solving situations of ELL students. The researchers formed two hypotheses: Simplified problems will result in more success and real world problems will result in more success, as well. Bernardo and Calleja kept prior research in mind when they used more simplified versions of problems for the ELL students (2005), and they focused on real life situations to see if students would use their background knowledge outside of school when solving word problems. The results of this study confirmed these hypotheses because the students rarely used background knowledge, even when the problems were given in their first or second language. The students were also more likely to use straightforward problem solving strategies, like they were taught, rather than use context outside of the school environment. This study brought about an important implication: that bilingual students are almost always taught mathematics and other core courses in their second language so the knowledge transfer is more difficult because of the language switch.

According to Johnston, Bottsford-Miller, and Thompson (2006), an instructional strategy might be using the think-aloud method. This method allows

students to engage in dialogue with themselves and/or other students and the teacher to help them process the information they are confronted with. The original study was conducted in order to improve the test design of large-scale assessments. The fact that students are discussing how they solve the problems allows the researcher to get a “better understanding of constructs, the student’s skill level, the relevance of items to student life experiences, and the relevance of items to the content taught” (Johnston et al., 2006). Even when students with cognitive disabilities, learning disabilities, or were English Language Learners, the addition of the think-aloud method was effective in their study.

### **Summary**

Almost all of the research points in the same direction and yields the same results, especially when looking at students from lower socioeconomic backgrounds. Previous research on vocabulary development has a direct connection to the research question regarding students’ lack of understanding and ability to problem solve in the mathematics classroom. Common themes that have been reoccurring among the research note the students’ inability to visualize what is happening mathematically. Another theme that emerged from the research was the students’ difficulty with the basic comprehension and understanding of word problems, and therefore their inability to correctly solve the math problem (Abedi & Lord, 2001; Bernardo, 1999). Not only do students need to read and comprehend the word problems they are given, but also another difficulty that

was noted was the students' struggle with being able to eliminate information that they do not need in order to solve the problem (Chapman, 2006).

An additional theme was the environment that the teacher and students created in the classroom. Classrooms that had a safe and comfortable environment created an atmosphere of risk takers and the students did not feel apprehensive when attempting word problems (De Corte et al., 2004), (Elbers & de Haan, 2005), and (McCrone, 2005). Where more traditional classrooms were viewed, the students were less likely to succeed in solving word problems (Morrone et al., 2004). A strong theme that arose from this area of classroom environment is the amount of time discussion was allowed and group work was structured within the typical school day. Adding in the dimension of English Language Learners, the research all agrees on the importance of dialogue, reading comprehension and if possible, the direct translation of word problems into the ELLs native language for success in mathematical problem solving (Martiniello, 2007; Bernardo, 1999; Bernardo, 2005).

## **Methodology**

### **Setting**

I am currently teaching at an elementary school located in eastern Pennsylvania. This year there are 371 students in grades kindergarten through fifth grade, 47% of which are girls and 53% are boys. 15% of our students qualify for free or reduced lunch. Our school has an intermediate unit room as well. There are six students who are English Language Learners, which is equal to 2% of our total population. The racial percentages are as follows: less than 1% are American Indian/Native American, 14% are Asian/Pacific Islanders, 3% are African American, 77% are Caucasian, and 6% are Hispanic/Latino. The community the school is located in is a suburban area adjacent to a small metropolitan city.

### **Participants**

My third grade classroom is comprised of 18 students, of which 42% are male and 58% are female. The ethnic breakdown of my class is 5% Hispanic/Latino, 63% Caucasian, and 32% Asian/Pacific Islander. 11% of my students are English Language Learners, but 28% of my students recently exited the English for Speakers of Other Languages program and are being monitored for the current school year and/or the following year. 11% of my students are eligible for free or reduced lunch.

## **Research Goals**

The purpose of this research is to develop dialogue and discussion strategies in order to increase understanding of problem solving techniques and mathematical understanding in my students. The large idea for this unit is the development of number sense using addition and subtraction. I also want my students to increase their self-confidence and motivation when solving word problems. Developing their mathematical and logical reasoning and improving their written responses to open ended questions were additional goals of my study.

I will be using data triangulation because of my multiple sources of data I will be collecting. I will be able to view my students' work, my documentation of discussion, and the survey and interview data to search for themes or inconsistencies. I can note what "works" and what doesn't when looking at the data in individual and small group environments. This will provide me with specific documentation as to student understanding and what level individual students are at when solving word problems. Through the use of the strategy of pictures, numbers, and words, I will gain valuable insight into how the students are processing and visualizing the information.

## **Data Gathering Methods**

### **Anecdotal Notes/Observational Field Log**

Data were collected on how often my students engaged in class discussions, what questions they were asking one another during the small groups, and how they responded to questions when solving word problems. I used this information to plan my instruction and tailor to the needs of my students in developing problem solving skills. I used observational notes through a double entry journal as well.

### **Student Work**

I collected student artifacts to use as data to measure individual student progress. Student work included number sentences with the operation missing, word problems that used prompts to guide the students through the process of how to solve them, logical and reasoning problems, and Sudoku number puzzles. I used a wide variety of curriculum materials, teacher created projects, and the strategy of using pictures, numbers, and words in order to reach the higher level thinking necessary to understand word problems. Having access to a wide variety of instructional materials provided me with the opportunity to use multiple intelligences, a variety of learning styles, and different skill areas to keep students improving their problem solving skills. I also used questions on the Study Island program (Appendix I) and other curriculum based assessments (Appendixes D, E, & F) that ask for students to explain their mathematical thinking as data for my

study. All of these measures were evaluated using a student friendly rubric on a four-point scale (Appendix J).

### **Student Interviews/Pre and Post-Survey**

The last type of data I collected was student interviews (Appendix H). I created open-ended questions in which I gathered more specific information than what was collected from the pre and post survey they each completed (Appendix G). Using all of the modes of student work, interviews and survey examples, and finally observation in a double entry will provide me with the triangulation necessary to form the most comprehensive look at what the students are learning and how they are learning.

### **Trustworthiness Statement**

In order to achieve trustworthiness in my study, I will use my double entry journal to document actual observations of the student interactions and discussions. Journal entries with quotations and reflective statements provide critical data when students are engaged in mathematical conversation. Participant feedback will be evaluated through the use of surveys and interviews throughout the study. The asking of open-ended questions will allow my students to explain why and how. Most importantly, student work will be collected and analyzed throughout the study making sure confidentiality of each of the participants is protected. All data collected for the use of the study will be coded with numbers and student pseudonyms while it is stored in a locked closet.

Even though I will be using a wide variety of tools for data collection, I will have biases when I implement this research strategy. I have learned to love math and really enjoy seeing children fully engaged in conversation and discussion when completing assignments that are just within their zone of proximal development. So, when completing my observational journal I made sure to keep observational data strictly separate from the reflection section on the other side of my double entry log. I remained as open-minded as possible and realized that surprises will surface throughout the study and that I would have to use them in order to improve my practice. All of the reading of professional books and analyzing of research studies dedicated to problem solving, collaboration,

vocabulary, and discourse provided me with a stable foundation in which I can refer to throughout my study.

I received approval with an expedited review of my research proposal from the Human Subjects Internal Review Board in June (Appendix A). After receiving notification of the acceptance of my research study, I sought approval by my principal through a signed consent form and discussion of my instructional plans and also received approval from my students' parents (Appendix B and C). All of the above reasons qualify my trustworthiness as a qualitative researcher.

## **Researcher's Story**

### **My Journey**

A hurdle necessary to reach my goal was producing activities that would provide differentiation and appropriate challenge for each of my individual students. During my pilot study and also my reflective seminar class I had created materials appropriate for the students I had prior to moving to another school. My students that I had taught for the past five years struggled in reading and comprehension and therefore had difficulty reading and understanding word problems in mathematics. I had developed materials necessary for the abilities of those students. I had to now focus my attention on the new group of students that I had this year, where reading and comprehending was not a major difficulty for them. Discussion, however, was a challenge for both of my populations, currently and previously.

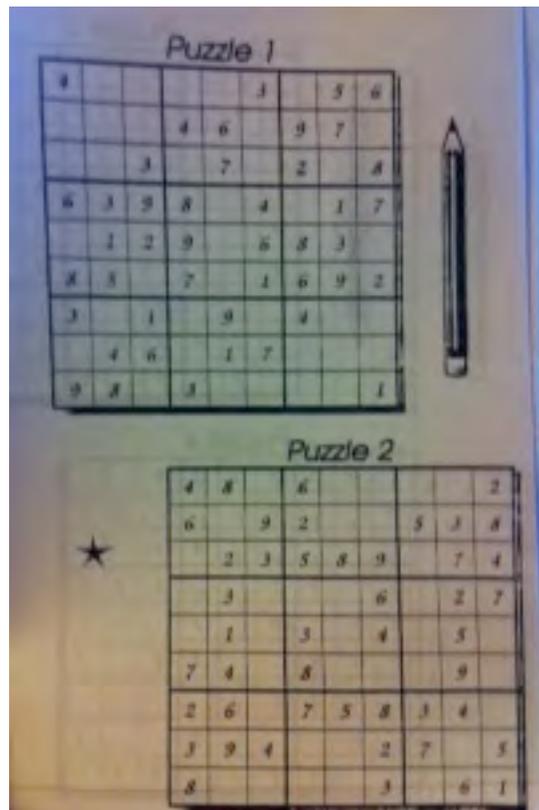
I began the study with mathematical objectives in mind and also thought of what my goals are for my students by the end of my study. I wanted my students to develop their communication and dialogue skills within collaborative learning pairs or triads. I also wanted them to improve their mathematical understanding of word problems, problem solving situations, develop higher level thinking, and improve their written explanations as to how they solved problems.

### **We're allowed to work together?**

Collaboration. The power of working together and using dialogue and discussion to help fellow students solve mathematical word problems. My students are accustomed to having individual accountability when completing assignments. My first challenge was to break them of this expectation and open them up to the new and different world of working in small groups, either pairs or triads, and completing group problem solving activities. All of a sudden my students' faces lit up with enthusiasm and excitement. As expected, the next question was if they could pick whom they could work with. At first, I thought that allowing student choice for grouping would increase their motivation, but keeping them focused to the point where each student was contributing was difficult.

I began my study by giving students a packet of problems to be solved, arranged from the least to the most complex. I told my students that they could solve the problems in sequence, or skip around through the packet. The first phase of my study involved Sudoku puzzles (Figure 1). The next series of problems required my students to use a chart to solve several single step word problems involving making change. The next few activities involved the use of guiding questions to help my students follow a sequence to direct them through the problem solving process of multiple step problems. This second phase was also where I introduced how to solve word problems using pictures, numbers, and

words. I then introduced using logic and reasoning to solve word problems and taught them how to use a matrix.



*Figure 1: The first page of Sudoku puzzles the students solved.*

To ensure my students were retaining how to use pictures, numbers, and words to solve a multiple step problem, I gave them a multiple step word problem next. I then gave them the cow and duck problem (Figure 2, pg. 45), which reviewed logical thinking and again used pictures, numbers, and words to solve the problem.

The third phase involved brainteaser activities, all leveled from basic understanding of single digit computation and number sense involving only addition and subtraction, to using double digits, and eventually multiplication and division number strings. The final and concluding activities involved all of previously learned skills with more difficult problems involving higher-level thinking.

### **Sudoku**

I decided to use Sudoku as a problem solving activity in the beginning of my study in order to get my students accustomed to talking and using dialogue with one another to solve logic problems. I began by asking the students if they have ever heard of or completed Sudoku puzzles. A few students raised their hands, but most had not heard of them. I started the puzzle by saying that we had to work together by speaking out loud. All students were expected to participate in the discussion and each student had to contribute to solving the puzzle.

I modeled a three by three box and showed them that the numbers one through nine must be in the box without repeating. I told them to turn and talk to their neighbors and decide how to go about completing this problem. A flurry of discussion occurred as expected. I jotted notes quickly as students rambled on about how easy this was going to be, that they should just go “one, two, three, etc. until all the numbers are used.” After about 30 seconds of discussion, the students looked at me puzzled. They wondered why this was called a puzzle because it was

too easy. I then told them that of course there is another step that makes this a true problem solving activity; they weren't allowed to repeat any numbers in an entire row or column.

I asked them to turn and talk for a second time and discuss possible numbers I could use to correctly fill in my puzzle. All students were discussing possible numbers to place in specific areas, and when one student posed a number that wouldn't work, another student would speak up and say that it wouldn't work because it didn't follow the rule of repeating numbers in vertical or horizontal form. They would then follow up with another possible number, waiting for group reactions to their answer. This was exactly what I wanted to happen! My students and I were so excited and they were highly motivated to try Sudoku in groups. I told them that throughout the next few months, we would be working on a variety of problem solving activities in pairs or triads and that I would be focused on their discussions about how they solved the problems. I also told them I would be grading their work as a problem-solving grade on their report cards as well as making sure they were held accountable for their contributions. Below is an example of a group of students engaging in dialogue where each of the students immediately understood how to solve the Sudoku problem, but had difficulty finding the solution.

*Adrian- wait we can try an 8*

*Max- count by one*

*Drake- ok guys, I'm stuck*

*Adrian- what can you put...*

*Thomas- let's try 6*

*Max- I see that*

*Steven- you can put a 5 right here*

*Steven- Sam, you're on puzzle 8!*

*Adrian- put a 7 or 8 here instead*

*Max- how about a 7 or 6*

*Adrian- something is wrong... you have to put a 5 right here*

*Drake- 3's hard, really hard*

*Max- puzzle 5 and 6 let's do them*

*Thomas- I'm trapped- I only need one more- I'm so close!*

*Steven- you can put a 5 right here*

*Steven- okay let me think*

*Adrian- you can put a 1 right here*

*Max- erase half of what you have and see where you messed up*

*Thomas- I'm getting a headache- I don't get this*

Looking back on this dialogue, I noticed that some of the students understood the process of solving the Sudoku puzzles, while others did not. Some of the students were solving the puzzles independently and just voicing to the

group members that they had solved it, and Drake was not even on the correct problem. This was one of the drawbacks to giving the groups several puzzles at one time and telling them that they get more difficult towards the end of the packet.

Having been with my students for a few months, I learned that many of them feel anxious when they receive a grade on class work or have to take tests. Using this information, I felt that having them in groups would allow them to let their guards down and work together to complete the activities. In order to collect data about student achievement, I would informally assess them as they were working, making sure they weren't aware. I continually collected student work and assisted the groups as they completed increasingly more difficult Sudoku puzzles.

I was very careful when helping out the groups so that I didn't give too much information away or give help to only one member of the group. This aided them in communicating with one another to solve the problem. Because I created the activities by adding more numbers in the beginning puzzles, and then gradually taking them away as the students experienced success, all of my students were able to build their confidence and use their voices to engage in problem solving. Also giving the opportunity to have groups choose their level of difficulty and allowing them to move to easier puzzles at their discretion added to their motivation. At a few points in the study though, instructions got

misinterpreted. Fortunately, my students were helping one another throughout and helped to correct these mistakes as evidenced in the example below.

*Christine- you just put random numbers*

*Lina- huh?*

*Jean-Marie- I think I need to start this whole thing over*

*Jessica- what if the 6 goes here- look, 5, 7, 6*

*Jean-Marie- gotta start with one box so there's a 1*

*Jessica- oh*

*Lina- it is a 3- I'm onto that row*

*Christine- this is not a 2 there's a 2 right there. But that doesn't work, can't be a 1, 2, 3, 4 maybe it's a 5- can't be a 6 wait, yes it can- 7, 8, no, 9 nope. Either a 6 or 5.*

My observations of these students' dialogue suggested that Christine, Jessica, and Lina understood the mathematical thinking needed to complete the puzzles successfully, although Christine did not realize this until the end of the dialogue. Also, Jean-Marie understood that when mistakes are made, a way to try to solve it successfully might be to begin the entire puzzle over again.

As I continued to write in my journal, I noted that my students couldn't wait for problem solving every day. They would ask throughout the day when it

was happening and if they could also work on the assignments at home. I wanted all work completed at school for obvious reasons, so after I answered that question there was a little disappointment across their faces. As the days continued, I monitored their progress and accuracy and recorded dialogue as quickly as possible. My students were asking for help less and less and only came to me when they had finished a puzzle and looked for feedback on how they solved it. At this point in my research I was looking for help on how to scribe all the dialogue, even with shortening words, making symbols, etc.; I was not able to keep up with the discussion. The next example was difficult to transcribe. These two students speak English as a second language and have heavy accents.

*Junee- if we put a 5 there we have to change 6? 8? No, not 8 over there 9? Yes! A 9 goes there!*

*Lina- Not a 3, is that a 9? You need to attach that*

*Junee- now I can't think...ahhh! What? If I put a 9 here, and a 7 here that would work*

*Junee- 7, 8, 9 this won't work I'm going to erase- first I'll finish that box then the other*

These two students work well together. They are both very timid and quiet students, but they are strong mathematically. They unfortunately are also meticulous when it comes to written work, so when letters and numbers are not

written perfectly, they must erase and fix it. I included this example because sometimes students are so focused on how the physical assignment looks rather than concentrating and putting their effort into the mathematical problem solving. When Lina focused on the proper writing of the number, she completely distracted Junee from solving the puzzle.

Some pairs or triads didn't work out as planned. In one triad, one child was trying to be the leader, but I taught my students that everyone must contribute to discussions. The dialogue below shows a group that had one leader and another strong student, but a third student who did not know where to begin. This example shows that cooperative learning situations can be difficult to implement.

*Taylor- we're missing numbers 4, 5, 8, and 9*

*I just figured out the answer- 2... can't*

*Jeremy- How do you know 1 doesn't go here?*

*Taylor- 1 goes where?*

*Jeremy- Why can't it go there?*

*Amy- I'm lost*

*Taylor- 1, 9... yeah I did it!*

*Eric- 6, 7, 8, 5, hmmm, Do we have it right?*

*Matthew- we already had that, guys, here...*

*Eric- show me this... let me see*

*Matthew- maybe 7 goes here...there! Uh oh- now we don't have an 8*

*Eric. - where would the 1 go? 1, 9, 6*

*Matthew- let Eric have it; he's our best person. It doesn't matter then, he can solve it*

This dialogue shows when collaborative groups do not understand how to work and function as a triad; they are working independently and then stating that they solved the puzzle. Eric was attempting to get the group on task and solving the puzzle as a group, but unfortunately Amy gave up and Matthew just assumed that Eric was the strongest student so what he says is the answer.

A few days later, our special education teacher's schedule had shifted and she came to me wondering if she could push in and support a few of my students during problem solving. I was so excited; this was finally the answer to allowing me the time to document all of the amazing strategies they were using to solve the problems. Each day I focused on one group and was nearly able to write down each student's contributions to the discussion. Even with her support, I was able to document each of the groups, but I know I wasn't able to write everything that was said during the activities.

Nearing the end of my first problem solving activity, all students seemed energized and enthusiastic about working on the Sudoku each day. My academically stronger students were nearing the completion of the puzzles that I

assigned them, so I knew it was time to begin to introduce the next section of my activities.

### **Cow and Duck Problem**

I began the same way as with the Sudoku puzzles, with a whole group introduction and some modeling. The next section of activities used logic and reasoning, just like Sudoku, but with reading and comprehension embedded in the problem solving activities. As a quick experiment to see what my students would do, I gave them the problem in Figure 2 below:

There were 15 heads and 48 legs on a farm. How many ducks and cows were there? Explain your answers using pictures, numbers and words.

*Figure 2: Cow and duck problem*

Wow, they just stared at me with disbelief! I allowed them to process the information through discussion with the whole group, myself included. I suggested that they begin by drawing what they know from the problem: the fact that there were 15 heads. We also knew there were 48 legs. Where the students had difficulty was trying to decide what to do next. I let them discuss within their groups how they were going to complete the word problem.

After 30 minutes, some groups were beginning to get an idea of what to do and others had solved the problem correctly. In order to push my academically stronger students further, I had them write number sentences and verbal

explanations to describe the mathematical process they followed in solving the word problem. This gave me time to go around to the groups that were struggling and give them support.

Developmentally, third graders are beginning to think in multiple steps and are ready to solve these types of word problems. They do need help though, and teaching them to draw a picture of what they are reading and understanding can assist them on what operations they need to use to solve the problems. I had begun the next phase of my study with the idea of using pictures, numbers, and words to help my students solve multiple step word problems.

I taught my students to approach problems three ways: through drawings, through number sentences, and through words. Drawing pictures of what they comprehend will help them understand what is happening in the word problem and it will also help me see inside their brains to understand how they are going to solve the problem. The number section requires the students to write number sentences to show what they are doing mathematically to solve the problem. The word section has the students explain the process of how they solved the word problem in sequence with a reason explaining why they did what they did, a justification. Below are some examples of my students' visual representations, number sentences, and verbal explanations.



Figure 3: Visual representation of cow and duck problem

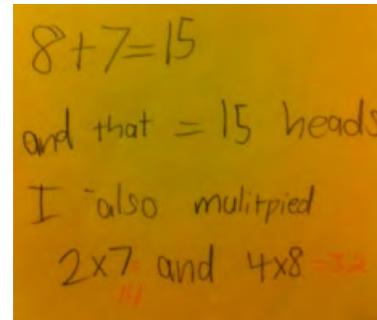
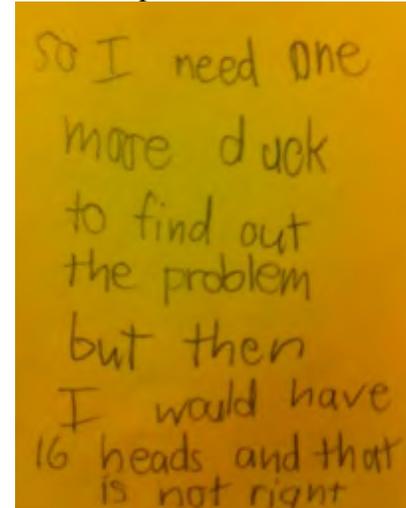
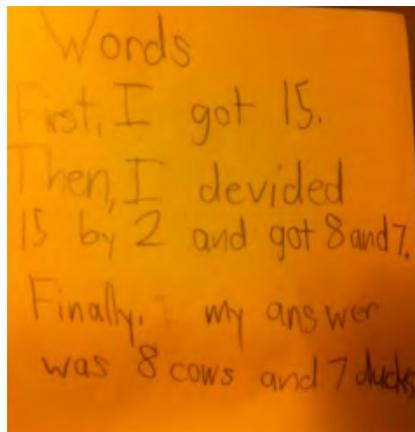


Figure 4: Number section of cow and duck problem



Figures 5 and 6: Words section of cow and duck problem

Looking at this students' work, I was able to recognize that some of my students knew that division means sharing between groups and also that they were using these groups to arrive at the solution of 48 legs.

My students were so excited when they solved this problem, but I continued to reinforce that these problems will take thought, time, and working in groups talking to solve them correctly. We continued to practice the pictures,

numbers, and words strategy by having the students working in collaborative pairs or triads to complete the problems in figures 7 and 8. These problems also involved logic and number sense to solve.

Beth's birthday is the first Monday in August. Sue's birthday is 20 days after Beth's. Kay's birthday is 2 weeks before Sue's. Dee's birthday is 7 days before Kay's. Whose birthday comes first and on what date?

AUGUST

S	M	T	W	Th	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

*Figure 7: Performance Assessment Calendar problem*

Alex and Andy ordered some Chinese food from this menu. Alex ordered won ton soup, 3 dumplings, and white rice. Andy ordered the same thing as Alex but also vegetables. Alex and Andy paid with a five-dollar bill. How much change did they get back?

Won ton Soup	25 cents
Noodle Soup	20 cents
Fried Rice	15 cents
White Rice	10 cents
Dumplings	30 cents each
Egg rolls	35 cents each
Vegetables	50 cents

*Figure 8: Performance Assessment Food Problem*

When I began documenting dialogue, there was a tremendous difference in the amount of dialogue between groups and who was talking in each of the groups. The academically stronger students were more vocal during this activity compared to Sudoku, where all students were able to participate freely. The stronger students all acquired the ability to process the information quickly and be able to verbalize it at a faster rate than my other students. When I would visit groups to observe the dialogue and manage their working habits, I would interject

so that each student was able to communicate. Ideally, I would have liked the students to do this automatically, but that didn't occur.

The following group of activities involved logic and using a matrix to solve the problem. I drew a matrix on the board with four columns and four rows. I then labeled each of the rows and columns and asked the students if they have seen this type of organizer used to solve a math problem. Only a couple of students had seen it, so I read the problem aloud, and began to fill in a few boxes based on the logic given in each of the clues (Figure 9). My students completed the guided model with ease and began to attempt the problems in the same groups as before.

1. There is a hyper-speed elevator in the brand-new Kookie Kola Koporate Headquarters Building. Going up it stops at the 3<sup>rd</sup>, 4<sup>th</sup>, 6<sup>th</sup>, and 10<sup>th</sup> floors. Each of four people gets off at a different floor. Bosco exits and says goodbye to Ermin. Siz is the last person to get off. Roz gets off one floor after Bosco. At which floor does each person get off?
2. Alma, Cornelia, and Hazel have just purchased the hottest new fashion accessory, Polygon Purses! Alma's purse has 2 more sides than Cornelia's and 3 less than Hazel's. If Cornelia's purse is shaped like a triangle, how many sides does Hazel's purse have? (Bonus: What is the shape of Hazel's purse?)

3. Alien explorers Blorg, Zatz, Klutz, and Peebleprax are standing in a row on the command deck of the Starship Bigsurprise. Klutz is not next to either Zatz or Blorg. Blorg is farthest to the left. Zatz stands next to Blorg. Starting from the left, how are the four explorers arranged?
4. The number of pieces of gum in Gussie's used chewing-gum collection is so secret, even she forgot how many there are. She does remember that the number of pieces is an odd number that is more than 100 and less than 200. The number is a multiple of 3 and 5. The sum of the digits in the hundreds and tens places equals 7. How many pieces does Gussie have in her collection?
5. Apprentice Wizard Walter can't remember which ingredient goes with witch (ha ha) spell. He found an ancient parchment with clues. Wolfbane is not used to turn an enemy to stone. Lizard tongue is used for invisibility. Batwing is not used for shrinking. Eye of newt is not used to cause sleepiness or to turn an enemy into stone. Which ingredient goes with which spell?

*Figure 9: Logic based sample problems*

The students completed these problems with ease, even though the challenge was there because they were written on an instructional fourth grade level. Using these data I was able to allow the students to attempt more challenging logic

problems without my support and in dialogue with their peers. The following activity illustrates what occurred next in my study. I wanted to see if they could apply logic, but as written in a more traditional word problem. It doesn't always go as planned though because the grouping of students may or may not be successful. In the problem below, what was meant to be a simple matching of color to vehicle type turned into a long discussion of how to combine colors and what new colors could be created.

### Word Problems

<p style="text-align: center;">Problem-Solving Strategy: Make an Organized List</p> <p><b>Problem:</b> Jan's favorite video game in his collection is <i>Fast Track</i>. To play it, he must choose a car, a truck, or a motorcycle to drive. He also has to pick the color for the vehicle—red, blue, green, or black. How many different combinations can Jan choose?</p> <p><b>Understand:</b> What will be combined in each choice?</p> <p><b>Plan:</b> If you start with a car, how will you make an organized list?</p> <p><b>Solve:</b> How can you check that you did not miss or repeat any combinations in your list?</p> <p><b>Solve:</b> How many different combinations can Jan choose?</p> <p><b>Look Back:</b> Why is car and truck NOT a possible combination?</p>
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*Figure 10: Problem Solving Making an Organized List*

These types of problems have scaffolding embedded in them in order to guide the students through solving single or multiple step word problems. The

first question guides students to look back into the problem and to locate the important information needed to solve the problem and then asks them to understand what the question is asking. The following steps have the students plan what steps they will take to solve the problem and check the answers that they came up with. The students are then expected to go back and revisit their solution and explain why an alternate solution that was given is not a possible answer to the problem.

*Ben: Ok so Jan's favorite video game in his collection is called fast track. To play it you must choose a car or a truck or a motorcycle to drive. We also have to pick a color for a vehicle: red, blue, green or black. How many different combinations can Jan choose?*

*Xavier: Well number one: what can be combined, what will be combined in each choice? So how about this, well, hmmm, well how about this? Let's get scrap paper and write the stuff we need to down and then once that's ready...*

*Ben: well it's red, blue, green or black how many different combinations? Well red and blue would make purple*

*Xavier: red and black would make black*

*Ben: red and purple would make green*

*Xavier: no purple, you can't do purple and green because it's already two things together*

*Ben: yeah*

*Xavier: Ok so what do we do?*

*Ben: wait*

*Xavier: so we can do red and blue which would be purple*

*Ben: exactly*

*Xavier: red and green would make god knows what,*

*Ben: just think red and green*

*Xavier: red and green just makes Christmas, just think Christmas*

*Ben: I think it would make*

*Xavier: cuz it can't make red, blue or yellow because those are the primary colors*

*Ben: exactly*

*Xavier: ok so red and blue and then blue and green make... Miss Eisenhard what does red and green make?*

*Miss Eisenhard: I don't know why you are combining the colors, reread the problem and make sure you are combining the correct things.*

*Both: OHHH okay!*

*Xavier: Okay now so how about car motorcycle? Car truck, you can't do.*

*Ben: Yeah*

*Xavier: so how about we do car motorcycle? And then what color?*

*Ben: what are we gonna do?*

*Xavier: so how about blue because you've seen those blue motorcycles and cars right? So maybe we can do it blue?*

*Ben: How about we just skip it and go to the next one?*

*Xavier: Just remember, it's gonna get harder*

*Ben: How about we go all the way to the back one? Trip Chips?*

*Xavier: we wrote down 8-6, is that the answer?*

*Ben: Yeah, didn't we finish this?*

*Xavier: Yeah 86 boxes! We finished the hardest one! Let's move onto Fruit Salad.*

*Ben: All right, let's do it.*

The pair of students did not solve the problem. They read the problem correctly, but made a mistake in what they were supposed to be combining. I told them to watch what they were combining in the problem, hopefully that clued them in that they were combining the incorrect things. Xavier began to solve it correctly, but when Ben suggested they move to the next problem, he gave up and they both moved to a more complex problem. This type of behavior occurred more often than I wanted it to throughout my study. It seems that when individual students felt frustrated, they would guess an answer and try a different problem whether it was more difficult than the one they had given up on or not.

The following problem was chosen to come next in sequence because it is set up similarly to the organized list problem. The key difference is the process in which the students could arrive at a correct answer. This problem is comparable to

the typical word problems my students see in their math textbooks, but without the guiding questions. The only part of this problem that my students may have difficulty with are the words: “twice”, “together”, and “reasonable”.

<p style="text-align: center;"><b>Problem-Solving Strategy: Guess and Check</b></p> <p><b>Problem:</b> A caterpillar has twice as many legs as a spider has. Together, they have 24 legs. How many legs does the spider have? How many legs does the caterpillar have?</p> <p><b>Understand:</b> Which bug has more legs? How do you know?</p> <p><b>Plan:</b> How will you check each of your guesses?</p> <p><b>Solve:</b> What is the solution to the problem?</p> <p><b>Look Back:</b> How can you check that your answers are reasonable?</p>
--

*Figure 11: Problem Solving Guess and Check*

*Jessica- okay let's go, you can check your answer by rereading the problem and check all the parts to make sure you are correct.*

*Christine- I'm doing something different- I'll do it a different way and see if we get something else.*

*Jessica- good... cool. Are you ready for the next page?*

*Christine- I can check and see if it is reasonable by reading the passage again and checking my answers.*

*Jessica- ok good and number 3?*

*Christine- Yeah and number 3 I wrote just in case that the solution is that a spider has 6 legs, but I thought wait, a spider has more than 6 legs.*

*Jessica- this is what I wrote, well I got 6 legs but then I knew that it would be 8 because a spider has more then 6 legs okay?*

*The solution is that a caterpillar has 16 legs and a spider has 8 legs and number 4 I can check it by reading and rereading the passage and check all problems and make sure you add correctly. Okay lets move onto the next page.*

*Christine- ok*

The problem below is an example of a pair of students trying to solve a logical problem and not understanding the vocabulary used in the guiding questions that are meant to help explain and assist the students in solving the problem correctly. The question that asks the students to understand the problem helps the students locate the four numbers they need to use to see which person jumped which length. The second question helps the students organize the information they were given in the problem. This is what the students had difficulty with. This example illustrates why sometimes a simple word used in a word problem can be problematic to the point where the students never arrive at an answer. They continue to contemplate what the words “table and logical reasoning” mean, when both of these students are capable at solving the problem correctly. In my observational notes, I wondered if instead of using these guiding questions as an asset to help the students, if they had inadvertently confused them instead.

Problem-Solving Strategy:  
Use Logical Reasoning

**Problem:** Jeremy, Erin, Mel, and Paco competed in the long jump. Their jump lengths were 9 feet, 10 feet, 11 feet, and 12 feet. Paco jumped the farthest. Erin jumped an even number of feet. Mel jumped farther than Jeremy. How far did they each jump?

**Understand:** What were the four jump lengths?

**Plan:** How can you use a table and logical reasoning to organize the facts?

**Solve:** How far did they each jump?

**Look Back:** Do all your answers match all of the facts given in the problem?

**Explain.**

*Figure 12: Problem Solving Logical Reasoning*

*Eric: Okay, so the first one...*

*Matthew: so the first one is 9-10-11-12.*

*Eric: Those are the feet.*

*Matthew: Use a table to...*

*Eric: and logical reasoning to organize the facts*

*Matthew: what does that mean?*

*Eric: Hmm let's think about it, hmmm I think it might mean what strategy would you use to organize the facts?*

*Matthew: Yeah, now that you told me what that means... what does that mean?*

*Eric: hmmm*

*Matthew: I'm not being funny; I really don't know what that means.*

*Eric: well we probably want to find a way to organize the facts, so what would we use?*

*Matthew: Ohhh!*

*Eric: how about you can draw a graph? That says feet on one side and then the name of every person on the other side and then it would, so then we could write how many...*

*Matthew: how many*

*Eric: how many feet they jumped*

*Matthew: okay*

*Eric: and it would be a four by one graph*

*Matthew: Actually I was thinking use a like, use a...*

*Eric: Do you think my idea is good?*

*Matthew: Yeah, ok, let's use that*

*Eric: ok*

*Matthew: no wait, I got it, you could use a table, yeah that's what it's called*

*Eric: you could use a table to decide how to solve this problem with the right number of feet on the side next to it. We would need to fill out the four by one grid. Do you think that's a good idea?*

*Matthew: Yeah, that works. How do you spell your?*

*Eric: Dude, we're partners, why do you have to write your?*

*Matthew: I'm making mine a little different.*

*Eric: Dude, I wanna see your answers to see if we are organizing our facts. How about by writing the names of each person on one side that has four, wait, that's what I think. Well that's what I think.*

*Eric: How about you just write I could draw a table with each person listed on the side and then write the amount they jumped on the other next to it. And the grid would be filled in; it should be a four by one grid.*

*Matthew: Let's just go on to number three.*

*Eric: Dude, you didn't write the answer, you have to write the answer. What you have doesn't make sense.*

*Eric: Okay, like I have an idea to help you. How would you use a table to help you find out the answer or organize the facts?*

*Matthew: Well I don't know what kind of table so I'm not sure. I'm thinking of a regular table.*

*Eric: A table is a kind of grid.*

*Matthew: Ohhh.*

*Eric: ok, so how would you use a table? Like that?*

*Matthew: Well I would use a four by four.*

*Eric: Well wouldn't a four by one, like this, be fine? Then you could write feet jumped at the top and then on the side you could write the names. Do you think that would be a good idea?*

*Matthew: Then who would be the person?*

*Eric: Look, here's an example. A four by one, look this is what I think it should look like. Because feet jumped that means this is how many feet they jumped. The other side shows their name, so in here is how many feet whoever jumped the amount.*

*Matthew: Ohhh!*

*Eric: So why don't you write that down?*

A challenge that continued throughout the study was time. My work was completely controlled by the clock, so students had to consistently start and stop each day while completing the tasks. This pair of students unfortunately never arrived at the correct answer without my support. Through observational notes I was able to jot down that they could not get past the language of the planning stage in order to solve the problem. So, the following afternoon I was able to tell them to cross out the questions and pretend they were not there to begin with. I gave them a new piece of paper with just the problem at the top and told them to try and solve it. They were able to recognize that they needed to draw a matrix and solve it like I had taught them earlier. Both Eric and Matthew were eventually successful.

The following is a different group of students completing the same problem.

*Jessica and Christine read problem together- ok it says, Jeremy, Ellen, Mel, and Paco competed in the long jump. Their lengths were 9 ft., 10 ft., 11 ft., and 12 ft.*

*Paco jumped the farthest. Jeremy jumped an even number of feet. Mel jumped farther than Jeremy. How far did they jump? Question number one says: What were the four jumps?*

*Jessica- Let's make a picture.*

*Christine- 9 feet, 10 feet, 11 feet, 12 feet.*

*Jessica- are you done yet?*

*Christine- no.*

*Jessica- this is what I wrote.*

*Christine- so we know that Paco jumped 12 feet because he jumped the farthest.*

*And...*

*Jessica- how can you use a table and logical reasoning to organize the facts?*

*Christine- and Erin jumped 10 feet. Since Mel jumped farther than Ellen, she jumped 11 feet. And then Jeremy then jumped 9 feet.*

*Jessica- good. Now, so how can you use a table and logical reasoning to organize the facts?*

*Christine- how can you use a table and logical reasoning to organize the facts? I don't know what that means?*

*Jessica- well maybe that means how can you like put them in a simple organized list? Maybe we can do like Jeremy, Paco jumped the farthest so maybe we could do like largest fact to smallest fact like the largest fact would be...*

*Christine- the largest fact would be Paco jumped 12 feet the next fact would be Mel jumped 11 feet, but then the next fact would be Erin jumped 10 feet and then the last one would be Jeremy jumped 9 feet. Hey, did you notice it went in order like the lengths?*

*Jessica- yeah it went in order by least lengths to greatest.*

*Christine- Yeah! Jeremy did 9 feet, Erin did 10 feet, Mel did 11, and Paco jumped 12 feet. So we could write...*

*Jessica- we could write like least length to greatest length. You can organize like the facts by putting your facts from least to greatest like this: Jeremy jumped 9 feet, Erin jumped 10 feet, Mel jumped 11 feet, and Paco jumped 12 feet. So Paco would jump the farthest.*

*Christine- I wrote you could organize the facts by listing the least things to the greatest things.*

Later on when I was examining this pair of students' work, I was able to assess that these two students did a nice job successfully answering the problem. They were able to decide what "organized list" meant to them, and just write the lengths of the jumps in order from least to greatest. A theme that often arises when students attempt word problems is the understanding of the language used and how it may interfere with the mathematical process. Misinterpretation, or if the students never climb over the hurdle of understanding the vocabulary, can

lead to students not completing the problem or never completing the mathematical process needed to answer the problem.

### **Number Strings**

The next phase of my study included a series of numbers where vocabulary and comprehension were not used, but number sense and logic played a huge role. I gave them a string of numbers and told them to use addition and subtraction signs between them in order to get to the solution listed after the equal sign. They used the guess and check method to solve the lengthy equations. The guess and check method involves the students trying an operation and looking at the rest of the numbers listed to try to get the solution. If the operations didn't work, they would make changes throughout to arrive at the end solution. The dialogue noted during this activity was different compared to the other activities. I wrote that a lot of the students relied on actually guessing which operations went where, not necessarily using their number sense to solve the number strings.

### Brain Teasers

Directions: Use different combinations of addition and subtraction symbols to solve the number sentences below. You will use the guess and check method to solve the sentences. Make sure you have an eraser! Good luck!

$$1. 6 \quad 8 \quad 2 \quad 4 \quad 3 \quad 1 \quad 5 = 23$$

$$2. 4 \quad 3 \quad 8 \quad 2 \quad 6 \quad 9 \quad 4 = 10$$

$$3. 8 \quad 4 \quad 1 \quad 0 \quad 5 \quad 6 \quad 2 = 6$$

$$4. 5 \quad 2 \quad 7 \quad 3 \quad 7 \quad 3 \quad 9 = 16$$

$$5. 4 \quad 2 \quad 8 \quad 6 \quad 1 \quad 0 \quad 9 = 28$$

$$6. 2 \quad 8 \quad 2 \quad 9 \quad 3 \quad 8 \quad 4 = 18$$

$$7. 5 \quad 1 \quad 8 \quad 3 \quad 6 \quad 2 \quad 5 = 12$$

$$8. 3 \quad 5 \quad 2 \quad 9 \quad 1 \quad 6 \quad 2 = 10$$

$$9. 2 \quad 8 \quad 1 \quad 7 \quad 2 \quad 8 \quad 4 = 26$$

$$10. 1 \quad 3 \quad 7 \quad 2 \quad 9 \quad 6 \quad 5 = 11$$

*Figure 13: Brain Teaser #1*

Because the students were mostly successful with the single digit number strings, I created a more difficult version, called Brain Teaser #2, involving double and triple-digit addition and subtraction.

### Brain Teasers #2

Directions: Use different combinations of addition and subtraction symbols to solve the number sentences below. You will use the guess and check method to solve the sentences. Make sure you have an eraser! Good luck!

1. 23 35 14 84 29 36 24 = 135

2. 49 32 176 27 97 276 89 = 256

3. 116 84 28 236 84 28 18 = 222

4. 86 178 189 142 136 286 187 = 254

Challenge: You can use addition, subtraction, multiplication, or division to solve this number string!

5. 4 6 8 217 188 8 12 = 48

*Figure 14: Brain Teaser #2*

*Taylor- okay, minus, minus, plus... wait...*

*Jean-Marie- wait, because if you subtract, 49 and 32 we get 7, I mean 17 if we add 32, to 176 we would get umm 143.*

*Taylor- 143*

*Jean-Marie- and then if we add 176 plus 227 it equals, we have to regroup, so it would be 149.*

*Taylor- so then what is, you said the minus 97 though*

*Jean-Marie- and then if we subtract that we would get 27-97, so we would cross out that make it a 1, cross out this*

*Taylor- we don't have to cross out! We can do 7...*

*Jean-Marie- How do you subtract? You can't subtract this. You can't subtract 2 and 9.*

*Taylor- yes you can! It's regrouping.*

I was so excited when my students were connecting what they were learning in their core math class to the problem solving activities, especially using the vocabulary they were taught.

*Isabel: Okay so Janice what did you get first? Ok so  $49 + 32$*

*Janice: First I added  $49 + 32 = 81$ , then I added  $81-176$  and I got 115 and then I added  $115 + 27$  and I got 142*

*Isabel: but what about if we get to a higher number?*

*Janice: uuuummm then we just try over,*

*Janice: Let's go:  $49 + 32=81$ , ...*

*Both: +, -, +...*

*Isabel: wait, wait so...  $151 + 27$  is .... Uuuuhhhmmmm ...*

*Isabel: 142*

*Both: 256... I got too big different number so I'm going to subtract right here....*

*Isabel: Ok ... we have to add, we have to add...*

*Janice: so we have +, -, +, +,  $142 + -7 =$  is 100*

*Isabel: is 239 is 239*

*Janice: ok, so first we do  $49 + 32 = 81$ ,  $176 - 115$*

*Isabel: not  $81 - 76 = 115$*

*Janice:  $115 + 27 = 142$*

*Isabel:  $142 -$  or  $+97 = 239$ ... that's what I have,*

*Janice: I subtracted that .... I subtract it when I did  $142 - 197 =$  ...It's supposed to get 256, ... I think I got it... so first +, -, +, +, -, +. Oh yeah great!!!...The teacher told me to do those too...*

*Isabel: Ok so +, -, +, wait, +, -*

*Janice: no, +, +, -, -, -, -, +*

*Isabel: wait, what was it again?*

*Janice: +, +, -, -, +, -, +, +, -, -, +, - I think that equals ... ok?*

*Isabel: Yep, now let's move onto number 4*

*Janice: All right, let's hear it.*

When some of my students experienced frustration, I would be there to help and guide them through the type of thinking they needed to do in order to solve the problem. One amazing observation was that only one group truly experienced complete frustration. They had started by just guessing, but then after many attempts, they didn't reach a solution. The other groups were experiencing successes after checking their work with me, which inadvertently frustrated that group more. The other students were doing it, why weren't they?

*Max-  $49 + 32 = 81$  so we should put a plus*

*Jenga- what next?*

*Max- well maybe we should put one more plus in between 32 and 176 and then a minus*

*Jenga- okay then a minus, then we should have a plus again or a minus?*

*Max- I would say minus*

*Jenga- okay, then plus, then plus again*

*Max- no plus then minus oh because if you are already at the hundreds you're going to get over this and you don't want to get over that so let's add to find out what do we have in all and see if we have 256 all together. So 49 plus 32 is 81 plus 176 = hmmm one hundred I mean two hundred sixty seven no I mean eight*

*Jenga- okay*

*Max- 60 or 57 – if we minus 27 it would like be 220 so if we subtract 97 more we need to regroup and think if we have 10 more, 10 that would be minus 97 more we would have...*

*Jenga- we should check our what we said right Theo?*

*Max- we would have about...*

*Jenga- 256!*

*Max- no 153*

*Jenga- but we need 256*

*Max- And then we're going to plus 176. So that means 276 more would give us*

*Jenga- three hundred some*

*Max- three hundred*

*Jenga- wait, actually four hundred something, four hundred*

*Max- three hundred 15 maybe I don't know. I think we should move on and try a different one and then come back to this one later. So...lets just skip that and go onto number three. So what we need in all is 222. So I should say we should*

*minus so we would have  $160-84 = 32$ . If we add 28 we have 60 so if we subtract 236 we wont have enough so we have to put a plus there then a minus. So let's*

*see, how much do we so far have? 296 so if we have 296 minus 84 we would have 212 and then we would put another minus we would have one hundred and 90...*

*Jenga- okay, but we need to have 222!*

*Max- so let's try to see how much now we have and think what to do next, plus 18 or minus 18?*

*Jenga- Max, Max!  $116 - 84 + 28 + 236 - 84 + 28 - 18$*

*Max- so how much would we have? Would we have 222?*

*Jenga- we should use a calculator!*

*Max- I just think that we need to think. First, we have 32 and then 60 then one more would give us 296 then 84-*

*Jenga- okay!*

*Max- then  $84 - 212$  - so if we subtract we're not going to have enough so how about we add and then subtract so now let's see if we had 80...*

*Jenga- that's what I said though*

*Max- we had 112 and then we add 28 we would have 130 so that would mean if we have 130 and we take 10 away and we have eight that means we have all 222!*

*Jenga- Nice!*

*Max- that means we have the just right amount! So the answer to number three is minus, plus, plus, minus, plus, minus. What do you want to do? five, four? Or two?*

*Jenga- I'm thinking we should do five. Here's what it says: you can use addition, subtraction, multiplication, or division to solve this number string. Oh, why would we do that? We should have four plus six plus eight that would be 18 plus 217-  $188 + 8$  minus 12 or multiplication*

*Max- so what do you think is the answer? Plus, plus, plus, minus, minus*

*Jenga- I thought we should have three pluses in the beginning one minus after the three pluses a plus after the minus and then a division sign for last.*

*Max- Division? I don't know how to do division. How would we know?*

*Jenga- let's ask Miss Eisenhard for a calculator*

*Max- So I don't think we can finish this by ourselves. So what would it mean maybe we should put multiplication then a minus? SO let's instead of a plus first how about we turn it into a multiplication? Multiplication is for the first number is how many groups the second number is going to be in. so four times six would equal – it would be four groups of six so six plus six is 12, six more is 18, and another six is 24. So if we have 24 we should put a minus I think.*

Up to this point, nearly every group had been successful. I chose to sit and work with Max and Jenga and guide them a little more. What I observed was that Max was trying desperately to solve the problem logically and was not getting the solution fast enough for the other student. Jenga would just guess, without using any kind of logic, and just assume that the solution would come sooner or later. I finally explained to Jenga that this is an activity that is going to take longer, that they won't arrive at a solution immediately. I thought that this student wanted to do the right thing and get the answer right, that he didn't remember that I was focusing on the process, not the product or end result. He probably is accustomed

to the teacher just looking for the students to arrive at the correct answer, not even pay attention to how the students get there in the first place. Eventually, this pair of students solved the problems with accuracy, but because the remainder of the class finished much earlier, I had given them extra time out of the normal routine to finish their work.

As the students were finishing this activity, I thought about how I could further incorporate their current objectives in math and connect them to problem solving. Our students were learning three digit addition and subtraction with borrowing and regrouping. I decided to use their learning in a second version of number strings where they would use the same method of guess and check, but incorporate addition and subtraction with three digit numbers. Because of the level of difficulty this posed, I only had them complete four problems and added a challenge at the bottom involving all four operations. Many of the groups were experiencing difficulty with this activity. I kept reinforcing that they had to try it one way, and when that didn't work, they should change one operation to see if it would work the other way. This pair of boys immediately jumps to the challenge problems when completing any of the previous problem solving activities, so this example is typical of this group. This example below illustrates the fact that several of my students are able to complete higher-level math even when they have not been formally taught these skills in school with our curriculum. Adding

in challenge problems created an option for my students who wanted to go beyond what was expected of them.

*Steven: Let's try and solve the fifth one. All right, so four + six = 10 times eight would be 80. Then 217, let me add that*

*Drake: 80 + 217 = 180*

*Steven: plus*

*Drake: 397 minus*

*Steven: we know four + six = ... ok let's write four times six*

*Drake: one, eight, eight*

*Steven: so four times six = 16*

*Drake: 17 - 8 = 2, 2, 9... yeah, we're not gonna get it. It's not division. It is not division.*

*Steven: 217*

*Drake: I just got 209*

*Steven: 24 + 217, 7 + 4 = 11, carry the one, so I have 241. It's 241-, so we have 53, 53...*

*Drake: Let's just add it all the way up to here. 217.*

*Steven: 53 minus 8 equals, equals 41, doesn't work. 53 + 8 = 61 -*

*Drake: 12*

*Steven: I think I got it!*

*Drake: It's not. Wait 49. Ahhh!*

*Steven: we're one away! Oh my gosh, we just got one away!*

*Drake: oh no, one away...*

An amazing revelation occurred towards the end of this activity. Some of the pairs/triads began to realize the relationships between operations. Because all of our students were taught fact families, and how numbers are related to one another using addition and subtraction, they had discovered that the same was true for multiplication and division. They had not formally been taught how to multiply and divide yet, but the students who were exposed to multiplication, helped the other group members learn how the operation worked. I had produced number strings as a challenge that involved multiplication and division, but on a very basic level. I used the numbers 100, 25, 75, 50, and 2 to see if my students would use money as a connection. As my students experienced success with those problems, I eased them into 12, 18, 24, 36, 48, etc. which had many factors associated with them. Because of this, many of my students were catching on and solving the more difficult number strings.

### Brain Teasers #3

Directions: Use different combinations of multiplication and division symbols to solve the number sentences below. You will use the guess and check method to solve the sentences. Make sure you have an eraser! Good luck!

1.     5                    4                    5                    2                    2 = 25

2.     6                    6                    12                    4                    3 = 4

3.     25                    4                    50                    36                    8 = 9

4.     8                    6                    4                    6                    2 = 144

Challenge: You can use addition, subtraction, multiplication, or division to solve this number string!

5.     43                    3                    15                    12                    8                    3 = 32

*Figure 15: Brain Teaser #3*

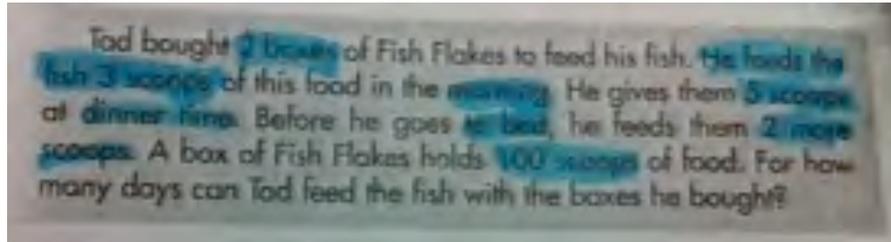
This was the third version of number strings, which I called brainteasers, and they involved only multiplication and division. I knew my stronger students would find this activity fairly simple because they already memorized basic multiplication facts, but I also knew that I had to build their self-confidence back up after they had really struggled with the second brainteaser activity. Almost all of the students were solving these with little challenge. Because most were getting

the strategies, I allowed the groups that understood a chance to help the other groups out. I split those groups up and had each student work with a new group and teach one another how to solve the problems. I told them to draw pictures to show one another how multiplication and division work. In no time at all, each group was successfully completing the third brainteaser.

Again, a challenge was placed at the bottom. The challenge involved double digit multiplication, but one where they could use repeated addition to solve the problem. I was in complete shock when some of the groups were able to solve it. I thought for sure I would be able to stump my strongest groups with this challenge. It did take them awhile to solve it, but the fact that they were successful was rewarding on both ends.

Nearing the end of this phase of my study, I was able to collect and analyze more data and found that my students did not need help in one step problems, nor did they need help in reading them. I knew that they would need help in multiple step problems. On previous chapter tests, I was able to see which students struggled in multiple step problems and what strategies they were using to solve the word problems. I was hoping that breaking up the word problems into parts and then solving each piece one at a time would help them become more independent and accurate when solving multiple step word problems.

## Multiple Step Word Problems



*Figure 16: Performance Assessment Fish Problem*

*Dexter- if he bought 2 boxes... counted by 10's to get the answer?*

*Jaylene- well ok... I don't think it's 120, 2 boxes- so  $100 + 100 = 200$  don't you think it will be more then? Because 10 days- he uses 10 scoops a day- how many days is our question...*

Another pair was completing the same multiple step problem using the same type of logic and arrived at the wrong answer because they doubled the number of days too many times. The fact that they had two boxes that was 200 scoops may have confused them. Matthew and Ben were on the right track up until the combining of the two boxes of fish food. Sometimes our beyond grade level math students have strong number sense, but often make careless mistakes because of their speed of their number fluency.

*Matthew- what's  $5 + 2 = 7$ ,  $7 + 3 = 10$*

*Ben- why are you doing that all over again?*

*Matthew- cuz there's 2 boxes- each are 100- it works*

*Matthew- first we added  $5 + 2 + 3 = 10$  don't forget the comma. Then we multiplied, how do you spell multiplied?  $10 \times 10 = 100$  2 times- twice- and we got 200. So he would have 40 days- wait, so ummm 5,  $10 \times 10 = 100$ , 10 days, 40 days...*

*Ben- Yeah, I bet, cuz if you multiply then add I would get the answer*

To have the students gain confidence when solving multiple step word problems, I intertwined questions throughout the problem to guide their thinking as well. I used two different published curriculum materials (Appendixes D, E, and F) that would help them through the process. I used a few of the problems as a pre-assessment to see which students needed what type of help on solving multiple step word problems. I had already taught them how to draw pictures, write number sentences, and then describe how they solved the problem in a word explanation. Using this as a form of assessment allowed me to see where my students were making their errors and which students already understood how to solve these problems. I used the data I had collected to work with my students who struggled the most. I guided them through the process of solving multiple

step word problems. The students were given several multiple step word problems to practice in their small groups that were used as assessment. I devoted a lot of time to the multiple step word problems and had used many different activities to ensure that my students had grasped how to solve these problems. I do have students that struggle with these problems, so I continued to work with these students throughout the school year.

Several of my observations in my journal showed that not every student was engaging in dialogue or having the chance to voice their opinions during the activities. I voiced these concerns to my classmates in my course and the idea of “talking chips” came up as a way to evenly distribute the chances to talk for the individual group members. After I had explained to the students what these chips were and the rules for using them, I had assigned each student five talking chips that they could use over the course of problem solving.

It was interesting how each group decided how to utilize them so that they would not run out of them by the end of the period. To help them learn how to manage the chips effectively, I would announce how much time was left in the period and also say about how many chips they should have left in equal intervals. Some students did not speak at all, they were too nervous that they would get to the end of the period and run out of chips so then they would not be able to talk. Other students used them too quickly and ran out too fast. After several days of practice and adjusting the amount of chips to a more appropriate amount, the

students were able to manage the talking of each of the individual student in the groups.

An important observation that I made during problem solving activities was that my struggling students were able to listen to my stronger students and apply the logic to solve the problems. They were learning from one another how to follow the logic. When groups came upon different solutions that did not match the others in the group, they talked about it as a group and decided the correct solution. When I noticed it, I provided positive reinforcement out loud to help enforce the expectations. Throughout the activities, I would continually remind them that they were completing fourth grade work. In order to push my students academically, I needed to give them higher-level work so that they were challenged.

### **Reinforcement/Higher Level Thinking**

The last phase of my study incorporated problems that caused all of my groups frustration. Most of my groups knew to draw a matrix to solve them, but they didn't know how to apply the logic given in each of the problems. Noting what my students were doing, I needed to take a step back. I had them use highlighters to locate specific words they needed to know the meaning of in order to solve the problems and also had them justify the size of the matrix they were using to solve them. The following set of problems involved students sometimes drawing a matrix, using money to calculate, logic and reasoning skills, and higher

level thinking skills to solve the problems. The order of the problems began easy and then became increasingly more difficult to solve.

### **Boardwalk Fun**

Colleen, Amy, Brian and Jimmy are headed for the boardwalk. Each plans to visit a different place first: DDR machine, frog game, bumper cars and race cars.

- Colleen and Jimmy do not have enough money for the car rides.
- Amy doesn't like fast rides.
- Brian and Jimmy think the Frog Game is impossible to win.
- Which place will each person visit first?


*Figure 17: Boardwalk Fun*

I taught my students how to decide how many rows and columns in their grid would be needed to solve the problems. Then using the clues and logic in the problems, we would fill in the grids with checkmarks or x's to match the clues and then solve the problems. These problems not only used logic, but used money, pictures, numbers, and words and most of the previously taught concepts

to reinforce their learning throughout the study. These two students below provide an example of successful collaboration and dialogue performed in solving a matrix problem.

*May: So you're saying do we actually do that?*

*Alexis: Well maybe Jimmy, maybe Jimmy wants to try and figure out the frog game.*

*May: I don't know*

*Alexis: Maybe we're supposed to figure that out.*

*May: well that wouldn't be like part of the question though cuz they are giving you clues and Jimmy doesn't want to like play the game.*

*Alexis: Because he thinks it's impossible.*

*May: Right. So that wouldn't be it. That wouldn't be right.*

*Alexis: Okay, so Colleen, I don't know if she wants to do the frog...*

*May: okay*

*Alexis: Let's mark Amy as the frog game. What did Miss Eisenhard say that the frog game was again?*

*May: I don't know, wait one second, let me go and ask okay?*

*Alexis: okay*

*May: It's the frog whack-a-mole.*

*Alexis: ohhh okay, it's the whack-a-frog.*

*May: sooo?*

*Alexis: so is that fast do you think?*

*May: I don't think so but you have to hit all of them when they pop up, so I think it would be fast because you have to do it fast. Amy can't have the frog game because it's fast.*

*Alexis: But I don't think she can have the dance, dance revolution either because that's what Jimmy has.*

*May: so this is kind of confusing but not really confusing right?*

*Alexis: okay wait, I'm gonna go ask...*

*May: wait, no what is the question?*

*Alexis: okay, no, nevermind*

*May: okay*

*Alexis: I think that we should put Amy in the frog game...*

*May: okay*

*Alexis: and...*

*May: okay, it's already in the frog game...*

*Alexis: and...*

*May: Where should Colleen go though?*

*Alexis: I'll be right back*

*May: okay, I wish I could come with you! Hmmmm, this is really confusing. If*

*Amy has the frog game nobody else can have it. But, ohhhhh, wait, I think that...*

*now I've got it. Race cars are bumper cars for Ryan and Amy but... I know it, I know it!*

*Alexis: what? Miss Eisenhard said...*

*May: I know it, but Brian can do either the bumper cars or the racecars if it, wait...*

*Alexis: May! Just hold on for a second...*

*May: sorry, I'm talking too fast*

*Alexis: Miss Eisenhard said that maybe that's a good question that Jimmy thinks it's impossible to win because Jimmy... we should try and figure it out*

*May: okay so we should erase Amy? Okay, so we're gonna put...*

*Alexis: okay so we're gonna put Amy...*

*May: Amy can't have the frog game, so...*

*Alexis: yeah, so...*

*May: ohh, ohh, well maybe we can put her in the d...*

*Alexis: dance, dance, revolution.*

*May: yeah, but what would Colleen get? I know right... but Colleen and Jimmy don't have enough for the car ride. SO that is really hard... cuz Colleen doesn't have enough for the car ride. So what are we supposed to do now?*

*Alexis: Hmmm.*

*May: Wait, I think I've got it! Alexis!*

*Alexis: what?*

*May: I think I've got it! Well, if, well, if Amy doesn't like fast rides, she has to go in DDR, yeah so she has to stay there. Then Brian can go either in the bumper cars or the racecars...*

*Alexis: yeah, ok, but I'm not really thinking about that right now cuz there's, wait a sec, hmmm. I think Jimmy was the DDR and Colleen played the frog game.*

*May: hmmm, this is pretty hard...*

*Alexis: well maybe we should just try and, what about Amy doing the bumper cars? They don't go fast! Then...*

*May: maybe, wait, that means Brian goes to the racecars! Yeah! We got it!*

Since this pair of students was successful in using dialogue to assist one another through the process of solving problems in mathematics, I continued documenting their dialogue when attempting more difficult problems. I wanted to see if my students would try to use the strategy of using a matrix to solve a problem that did not need it. Baseball season is a problem where the students needed to make a list, use a table like a t-chart, or simply draw pictures in order to accurately solve.

### Baseball Season

John is trying to get into shape for baseball season. He runs 1 mile on the first day. He runs two miles each day for two days. Then he runs 3 miles a day for three days.

- If John keeps following this running schedule, on what day will he reach his goal of running 5 miles in a day?
- How many miles will John have run altogether when he finishes that 5-mile run?

*Figure 18: Baseball Season*

*Alexis: Baseball season. Draw the matrix. Wait, I don't know that this one needs a matrix.*

*May: Well we still want to cuz it's easier.*

*Alexis: wait, he runs one mile on the first day, he runs two miles each day for the next two days, then he runs three miles a day for three days, if John keeps following this training schedule, on what day will he reach his goal of running five miles a day? How many miles will John have run all together?*

*May: wait! We should actually draw the matrix.*

*Alexis: well how would that help us?*

*May: it would probably help us easier, probably help us better*

*Alexis: only four though? I don't know I just did that. Oh! We have to do five.*

*May: Okay! So we gotta erase that and put that down lower... there that's better.*

*It's 3-D; its not 3 side by side. Its what I forgot.*

*Alexis: four fives, so we have to put that down there. Ok so...*

*May: This one's hard too!*

*Alexis: Well she said it would get harder...*

*May: as we go.*

*Alexis: Yeah, so, hmmm. It's just so tough! Well I didn't think so at all!*

*May: Man, this is hard.*

*Alexis: I think we should, May, I think maybe we should just take off the one day, we should probably do seven days...*

*May: Seven days a week! Well, maybe I don't know how many lines I need I just did that. Two, three, four...*

*Alexis: Okay, so for one day, for one day he did it, wait, one second. So for two days he did this. And then he does it for three days.*

*May: AHHH! Man this is tough!*

*Alexis: no it's not really when you figure it out! Oh so you just wanna check, check, check...*

*May: Yeah! Check, check, check, and then he does four.*

*Alexis: But wait, maybe we won't need all this. 1, 2, 3, 4, 5.*

*May: Ok, so 1, 2, 3, 4, 5.*

*Alexis: If we were doing this by, if he started on Sunday, then he would end on a Sunday.*

*May: yeah, if it just started like that...*

*Alexis: So how many miles will John have to run...*

*May: Yeah, but what day does it start on?*

*Alexis: I'm gonna ask her.*

*May: ok, so now we get it!*

*Alexis: ok, we have to add it all together.*

*May: So, we add: one day and then two days and then three days and then four days and then five days...*

*Alexis: One, two, two, three, three, three, four, four, four, four, five...*

*May: We can make a big number strip*

*Alexis: we don't need to...*

*Alexis: just add them together*

*May: okay, so three, five, eight, fourteen, four more fours is 12 more, 26 and then five is 31- ahh I forgot my highlighter! Can I have the highlighter real quick?*

*Alexis: yeah, sure. I think we're finished with this one, but hang on...*

*May: yeah?*

*Alexis: I don't think 31 is right*

*May: ummm why not?*

*Alexis: cuz, if we look at the problem again, it says that he has to run that last five mile day. So 31 plus another five. No, wait, it's 30*

*May: 30? Huh?*

*Alexis: yeah, cuz we didn't add right. It's 30 plus five equals 35.*

This pair of students did exactly what I thought they would do by trying to use the matrix to solve the problem. But what excited me more was that through dialogue they were able to realize that they did not need to draw a matrix to solve the problem correctly.

The next problem used money that the students were learning in their core math classes, but again required that students draw a matrix. The terms “fewer”, “more”, “fewest”, and “equal or same number” were used and previously challenged my students in their mathematical understanding. The clues in the first two paragraphs also pushed my students to use higher-level thinking and logic.

### **Problem Solving Brain Teasers**

#### **Change Range Question**

Rosa, Glen, Cathy, and Errol each bought snacks from several vending machines. Each person got back 6 coins, but they were different combinations of coins.

Each person received less than \$1 in change. The machines returned only nickels, dimes, and quarters. Each person had at least one of each coin.

- Rosa had the fewest dimes but the same number of quarters as Errol.
- Glen had an equal number of nickels, dimes, and quarters.
- Cathy had the same number of quarters as Glen.
- Errol had more dimes than Cathy.
- Cathy had more dimes than quarters.
- Rosa had fewer quarters than Cathy.

How much money did each person get back from the machines?

How much money did they get back in all?

*Figure 19: Change Range Question*

I wanted to see if my proximity near this group of students would affect them when trying to solve the problems despite their frustration. Up to this point, the students would get very close to solving the problem, but the one student would not know which direction to go in and would give up. This frustration would take over and ultimately both students would move onto another problem, even if it was more difficult.

*May: Hold on, ok, so now we can go onto the change range problem. That looks hard! But, it might be easy! Cathy and Errol...*

*Alexis: wait for me! Ok, Rosa, Glen, Cathy, and Errol each bought snacks, each bought snacks from several vending machines. Each person got back 6 coins but they were different combinations of coins.*

*May: Oh!*

*Alexis: Each person received less than \$1 in change. The machines only returned nickels, dimes, and quarters. Each person had at least one of each coin. Rosa had a...*

*May: Wait, should we draw a matrix? Maybe!*

*Alexis: Ohhh, wait, we should, 1, 2, 3, 1, 2, 3, Rosa, ok, Glen, Cathy, and Errol. And then we need nickels, dimes, and quarters, ok so!*

*May: Wait!*

*Alexis: Ok, Rosa had the fewest dimes, but the same number of quarters as Errol.*

*May: So there would be hmmm, well that's just hard. But we know that...*

*Alexis: Wait, Cathy has, wait, the same number of quarters as Glenn. So we need to figure out how many Glenn has. Glen has the same number of, an equal number of nickels, dimes, and quarters. So, does that mean that all of them added together is an equal number? Or all...*

*May: Didn't it say 60, or six coins, but they are a different combination.*

*Alexis: Hmmm*

*May: This is a brainteaser?!?!? It might not be what you think it is! But if we think it's something it might not be what we think it is. I know what I said is confusing!*

*Alexis: Glen had an equal number of coins,*

*May: but Rosa had the fewest coins, no had the fewest dimes, but the same number of quarters as Errol. Oh ok, so the fewest dimes, so we know she has a dime.*

*Alexis: but maybe, I'm wondering if we should maybe write the number? Maybe we shouldn't do it this way. Maybe we should just skip it!*

*May: Yeah! Skip it and come back to it!*

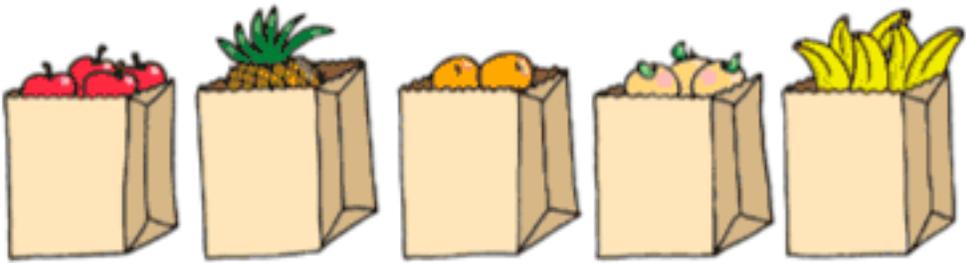
Again, allowing the students to freely move between problems presented challenges as well as successes. The students were on the right track when they

began with Glen, they even reread the clue that gave them the hint that each person had six coins. Since Glen had three different types of coins, they would have figured out that he had two of each coin. They were able to recognize that they needed to draw a matrix to solve the problem.

The next problem that is more difficult than the last, presented another set of challenges for my students. They needed to use multiplication, the clues in the pictures, and other multiple steps to solve the problem shown below.

**Fruit Salad**

**Question**



4 apples    1 pineapple    2 oranges    3 peaches    6 bananas

Fran, Gus, Harry, and Kim bought 3 bags of fruit each.

- Fran bought 2 bags of apples and one bag of oranges.
- Gus bought the same number of oranges as Fran, and the rest of the fruit he bought were peaches.
- Harry got the same kinds of fruit as Fran but a different number of each fruit.
- Kim chose fruit that no one else bought. She bought two kinds of fruit. She had more than 10 pieces of fruit in all.

How many pieces of fruit did each person buy?

*Figure 20: Fruit Salad*

*Junee: Okay, so four different people and five different kinds of fruit.*

*Lina: maybe we should draw a matrix like Miss Eisenhard taught us?*

*Junee: I don't think we can...*

*Lina: why not? We have information to fill it out... and we're matching up which person buys which fruit*

*Junee: yeah, but the first part says Fran bought 2 bags, the picture only shows one bag...*

*Lina: hmmm, I'm not sure what that means, but what if each bag had four apples, the next had one pineapple, each bag had 2 oranges...*

*Junee: yeah, Lina! I think that's it! Cuz the first clue says 2 bags of apples, so that would be eight apples.*

*Lina: ok, so Fran bought eight apples and two oranges.*

*Junee: we need to do the second clue now. Gus got two oranges cuz it's the same as Fran and then peaches, but it doesn't give us a number.*

*Lina: maybe we should read all the clues first, then try to solve.*

*Junee: okay, so Gus bought the same number of oranges as Fran, and the rest of the fruit he bought were peaches. Harry got the same kinds of fruit as Fran but a different number of each fruit. Kim chose fruit that no one else bought. She bought two kinds of fruit. She had more than 10 pieces of fruit in all. How many pieces of fruit did each person buy? That just confused me more Lina!*

*Lina: wait, it says that they bought three bags of fruit each by the picture. So, he must have got two bags of peaches, cuz he bought a bag of oranges.*

*Junee: okay, so...*

*Lina: so... that would be three + three is six. Six peaches.*

*Junee: how do we put this in a matrix?*

*Lina: umm I don't know. Let's just keep going...*

*Junee: Harry got the same kinds of fruit as Fran, but different numbers of fruit. So I'm guessing he got two bags of oranges and one bag of apples.*

*Lina: how did you know that?*

*Junee: cuz one plus two is three and two plus one is three, too!*

*Lina: ohhhh ok... so four oranges and four apples.*

*Junee: yeah... but Kim chose fruit that no one else got. And she got two different kinds and more than 10 pieces of fruit.*

*Lina: but I thought they had 3, how can she have 10???*

*Junee: cuz Lina, it says pieces not bags!*

*Lina: ohhh okay. So she got pineapples and bananas. Peaches, oranges, and apples were already used. Hmmm this is hard...*

*Junee: We're almost done, come on Lina! Pineapples, ummm there is only one in a bag. But six bananas. 10 pieces of fruit.*

*Lina: and it has to be 3 bags*

*June: what if she buys two bags of bananas?*

*Lina: that's not right, that's 12, it can't be more than 10*

*June: okay, so only one bag of bananas.*

*Lina: two bags of pineapple! Yeah! That's 10!*

*June: wait! No!*

*Lina: what???? It says more than 10 pieces! So it has to be two bags of bananas which is 12 and 1 bag of pineapples which is one. 13 pieces, that's more than 10!*

*June: okay, we did it...*

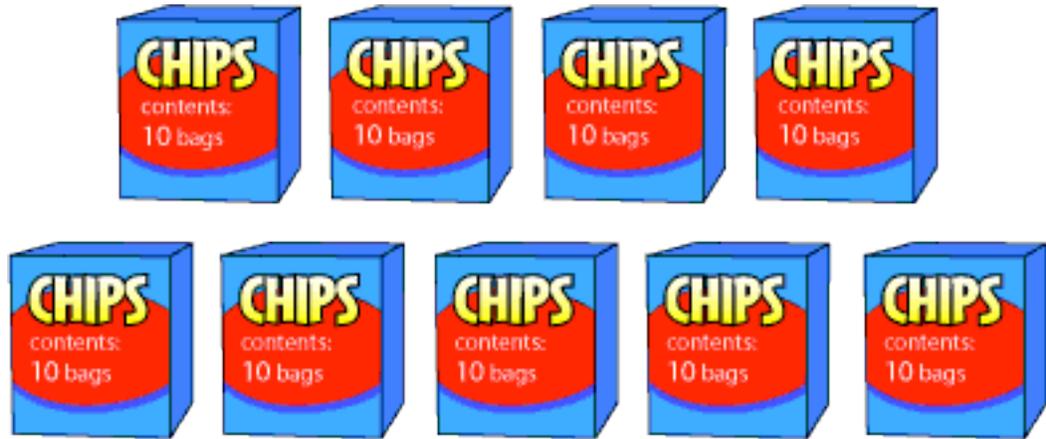
*Lina: that was a hard one!! Yeah, Miss Eisenhard said the ones in the back are the hardest and I think we got it!*

I thoroughly enjoyed this amazing dialogue between Lina and June. They truly progressed throughout my study to a point where they began to feel comfortable using dialogue. Initially, they barely spoke unless called upon in class. Each student was contributing to the discussion with important mathematical thinking and problem solving and both of them were able to understand how to solve one of the most difficult problems used in the study.

The Trip Chips problem shown below was the most difficult problem I had the students solve. They needed to use multiple operations, information in the picture, clues, and also understand specific vocabulary used in the problem to solve it correctly. This problem required the students to use the nine boxes of chips and multiply that number times the 10 bags that were in each box. The problem uses the phrases “fewer” and “twice as many” which confuses many of my students. The trip chips problem below is a multiple step problem that poses several mathematical processes to occur in order to solve it correctly.

## Trip Chips

### Question



The scouts packed 9 boxes of different kinds of chips for their trip. Each box had 10 bags of chips. These clues show what they had left at the end of the trip. They ate all the rest.

Clues:

- 2 full boxes of potato chips and 3 bags left over
- 1 fewer box of nacho chips than potato chips, with no bags left over
- twice as many loose bags of barbecue chips than loose bags of potato chips, with no full boxes left over

How many bags of chips did the scouts eat on their trip?

*Figure 21: Trip Chips*

*Ginka: Jose, I don't get the umm trip chip problem because I tried it and Miss*

*Eisenhard says its wrong, so could you help me?*

*Jose: umm I don't know either, but let's figure this out together. All right? So,*

*how many bags...*

*Ginka: The scouts packed 9 bags of different kinds of chips for their trip. Each box has 10 bags of chips. These clues show what they have left at the end of the trip. They ate all the rest.*

*Jose: Alright- 2 full bags of potato chips and 3 bags left over. One fewer...*

*Ginka: One fewer bag of nacho chips than potato chips with no bags left over.*

*Jose: twice as many of those bags are barbeque chips and there are no bags left over. How many bags of chips?*

*Ginka: Well I thought they ate 23 bags of potato chips, 13 bags of nacho chips, and 33 bags of barbeque chips because...*

*Jose: well let's see...*

*Ginka: ummm*

*Jose: well there's only so many bags of chips you can choose from and there's not much*

*Ginka: and there's only 3 kinds of chips*

*Jose: so you can't have that many kinds of chips if there's only so many kinds of bags, are you following me?*

*Ginka: umm, no. I don't really get that.*

*Jose: all right, so. There's only a couple bags of chips right? Only a couple of bags like 9 bags of chips.*

*Ginka: yeah, 9 bags*

*Jose: but if they can eat, wait did they say how many chips or how many bags?*

*How many chips or how many bags? Because if it's how many bags like they can only eat 9 bags, but if it's how many chips...*

*Ginka: wait, no, no, no... maybe there's bags, there's 10 bags in umm a box so maybe they ate 23 potato wait, they have a box, 2 boxes, and they ate...*

*Jose: equals 28 I mean 23 umm*

*Ginka: yeah and then 7 left in one box, so 13, so maybe they ate 17 no maybe they ate, yeah 17 bags of nacho chips since there's 7 left in one box and there's 10 in one bag. So I think there's 17 nacho chips.*

*Jose: okay, but if there's 17 nacho chips how many nacho chips would – if they had 17?*

*Ginka: no they-*

*Jose: so it's not 13, it's 17 right?*

*Ginka: yeah*

*Jose- yeah so it looks like we're done...*

*Ginka: no, no, no, no! Barbeque chips, so they ate 17 so two boxes 1, 1, 1 so they ate 3 boxes right now.*

*Jose: yeah*

*Ginka: and they umm I think there's 33 bags of barbeque chips but 33 bags equals 3 boxes and one more box with 3, 7 left in it.*

*Jose- all right, so*

*Ginka: so*

*Jose- so 30 could be 7? Or 7...?*

*Ginka: no, wait*

*Jose- what are you trying to tell me?*

*Ginka: I think the answer would be this: 23 bags of potato chips, 17 bags of nacho chips and 33 bags of barbeque chips.*

*Jose: yeah, that's probably what it is so- and since he's got it right so he's got the answer so I think yeah, that's the answer.*

*Ginka: oh yeah, what did you need help with again?*

*Jose: I needed help with this. 23 bags of potato chips, okay, and 17 bags of nacho chips, yeah, and 33 bags of barbeque chips.*

Using this dialogue, I was able to collect important information about the mathematical thinking this pair of students was engaging in. Jose attempts to understand the clues by reading them aloud, but ultimately he does not engage in any mathematical discussion. Ginka begins with 20 bags of potato chips, but does not use the term “left over” correctly, he added instead of subtracted. He then followed with nacho chips and reused the term “left over” and added another three. He is very close with 33 bags of barbeque chips, but it should ultimately be labeled as 33 bags of chips eaten. What is missing from this dialogue is what Ginka did mathematically to arrive at the correct answer of 33 bags. He multiplied

nine groups of 10, subtracted 17 potato chips, subtracted 10 bags of nacho chips, and subtracted 6 bags of barbeque chips for a total of 57 bags left over. He then took his original 90 bags and subtracted the 57 bags left and correctly solved that there were 33 bags eaten on the trip.

Because Jose was not engaging in mathematical dialogue, I also showed them the process I was using to get them started on solving problems in their groups. I wanted to make sure they were discussing and questioning one another on the process of solving them. Eventually, some of the groups were solving the most difficult problems, but again, my struggling students were solving them up to a certain point and then experiencing frustration. I kept pushing them back into their groups forcing them to use discussion and only giving them a little help so that they would not give up.

Throughout the study, I was conducting one-on-one interviews asking my students what they enjoyed most and least about math and problem solving and how they were going to use what they were learning in problem solving in real life in the future. As the end of the study was approaching, I gave my students the post-survey and analyzed the results. I also assigned independent post-tests, one focusing on a variety of single and multiple step word problems. The other test required that the students complete a multiple step word problem and use pictures, numbers, and words to explain how they solved the problem (Appendix K).



## **Methods of Data Analysis**

A variety of data sources were used throughout the course of my study. Using at least three different resources allowed me to view my teaching and the students' understandings from multiple perspectives. As I collected and reviewed the sources, I was able to make changes in my instruction, evaluate and measure student understanding and student learning and alter student assignments as well.

### *Surveys*

A pre-study survey was given to individual students to complete in the beginning of my study (Appendix G). It was a 17-question survey that was read aloud to ensure student understanding. My students needed to understand the differences between the vocabulary terms of never, seldom, sometimes, usually, and always. The survey was given in order to collect the students' feelings and attitudes about themselves towards math, how they learn best, and what types of things need to happen during math so that they learn. Identical to the pre-study survey, a post-study survey was given as well to measure any changes of attitudes or feelings after the conclusion of the study.

### *Interviews*

One-on-one interviews were conducted throughout the study as well. I asked open-ended questions where I expected the students to explain their answers from the survey in a more detailed fashion. The students explained what

they like and dislike about math and word problems. They also explained how they would use their mathematical learning in the future.

### *Student Work*

A wide variety of student work was collected throughout the study. I collected Sudoku puzzles, several single and multiple step word problems, logical/reasoning problems, and problems involving computation and number sense in a problem solving context. All work began at a basic cognitive level and built upon previous understanding and continued to more cognitively demanding problems. I also was able to use word problems that had questions built-in to scaffold the learners on how to process the language in the problem. I also was able to use the Study Island website to collect a beginning of the year baseline level for students' math understanding as well as a middle of the year level. This benchmarking assessment tool has a high correlation with how the students will perform on the PSSAs.

A post-test assigned through the Study Island program was administered. The first post test consisted of 12 word problems, 6 single step and 6 multiple step problems. The other post test measured the students' written explanations of how they solved a multiple step word problem using pictures, numbers, and words. (Appendix J). The results of the word problems post test were: 56% of my students correctly solved twelve problems, 22% correctly solved eleven, 11% had two wrong, and 5% of the students solved eight correct, and the other 5% solved

seven correctly. The post-test involving the use of pictures, numbers, and words to solve a multiple step word problem was less successful. The results are as follows: 50% scored advanced, 28% scored proficient, and 22% scored on a basic level.

### *Field Log*

A field log was chronologically written throughout the study to capture as much dialogue as possible between the groups of students. I also included a reflection section as well as a coding piece to document how activities, dialogue, an environment intertwined with one another. The reflection and dialogue section formed my story on how the study occurred. The coding piece allowed me to see recurring statements or moments where connections were made and how often they occurred. I then used these codes in the graphic organizer and then in sentence form to make theme statements.

### Theme Statements

- *I have evidence that encouraging dialogue between students in cooperative learning groups when solving mathematical word problems helped my students to increase their motivation and self-confidence.*
- *I have evidence that drawing pictures and engaging in dialogue when solving mathematical word problems helped my students to improve their depth of mathematical understanding.*
- *I have evidence that encouraging dialogue between students in cooperative learning groups when solving mathematical word problems helped my students to strengthen their higher level logic and reasoning skills and number sense.*
- *I have evidence that encouraging dialogue between students in cooperative learning groups when solving mathematical word problems helped my students to improve their written skills.*

The coding that I had completed throughout the field log, student work, interview and survey data were compiled in the form of bins. The following is a diagram of what I had visually created for the recurring themes.

What are the observed and reported experiences when student dialogue and group work are implemented within a third grade problem solving mathematics class?	
1. Student Motivation and Confidence	Engagement, attitude, work ethic, management, collaboration, body language, achievement, scaffolding, challenging activities, tiered/leveled activities, activities are progressively more difficult, emotional intelligence, teacher influence
2. Depth of Mathematical Understanding	Assessment, observation, expectations, modeling, achievement, strategies used, application of skills, visual representations, scaffolding, strengths and weaknesses, different ability levels of students, activities are appropriate levels
3. Higher Level Logic and Reasoning Skills and Number Sense	Metacognition, purpose, mastery of skills, achievement, application of skills, scaffolding, tied to curriculum
4. Verbal and Written Communication Skills	Accountability, interaction, focus, achievement, visual representations, scaffolding, student and friend influence, pictures, numbers, and words

Figure 23: Bins



Figure 24: Visual representation of coding terms

## Findings

I began my study with strong motivation for engaging students in higher-level problem solving activities using dialogue in collaborative learning groups. Using the objectives, curriculum strategies, and the NCTM standards I developed activities that would compel my students to use verbal communication as a means to solve mathematical situations. I have a group of students who are above average, a group of average, and a very small group of struggling students. I also observed my students during social situations, group work, independent work, and various other contexts.

Using my teaching tools, assessments, observations, and my pre-survey about attitudes and perceptions of mathematics, I began my study. I was hoping to increase motivation when solving word problems, build my students' self confidence, improve their mathematical understanding, and engage them in higher level activities that would strengthen their number sense, logic and reasoning abilities, and improve their verbal and written communication in the mathematical setting.

- *I have evidence that encouraging dialogue between students in cooperative learning groups when solving mathematical word problems helped my students to increase their motivation and self-confidence and develop their autonomy.*

Students were given the opportunity to choose varying levels of problem solving activities ranging from simple to complex. They were also allowed to move freely between the activities when they felt frustration or unsettled with their answers. This allowed my students to develop autonomy and work in cooperative learning groups at their own pace. Adrian said during the student interviews that he likes when word problems are challenging, but he doesn't like when they are too hard. Drake also commented on the difficulty of word problems, but he enjoys the fact that he can try problems that are "as hard as I want." A question in my survey asked if my students enjoyed challenging math problems and 72% of my students either chose "always" or "usually" when they answered that question. As noted in my field log, I would continue to remind the students that many of the problems they were solving were on a 4<sup>th</sup> grade level, so I expected them to be challenged. When I would show a look of shock and amazement on my face when students began solving the more difficult problems, the remainder of the groups would react with enthusiasm to try them as well.

All activities were using what the students were learning in their core math classes using flexible grouping. Using the classroom data as a guide, although I allowed the students to choose whom they would work with, I was the ultimate authority if there were groups that would not function properly. Ella remarked in her interview that "working in groups is what I like most in math because you can help people to solve the problems."

Often throughout my field log, I noted that students were motivated to complete the activities through my observation of their facial expressions and body languages. There were many times throughout my study where the students could not wait until I reminded them of specific strategies to use and forced me to cut my explanations short so that they could begin the activities. I found myself writing that I need to keep my introductions as short as possible because most of my students did not need the reminders every three days. It's almost as if my students were telling me through their body language and facial expressions that "we get it Miss Eisenhard, just let us get to work!"

Competition between the groups also was observed throughout the study. Once students began checking their answers with me and hearing that they were close, or almost there, or not quite, other groups would listen in and want to be the first group to solve the challenging problems correctly. Groups were continually trying to get the answers right so that they could move on to the more difficult problems. Students were given the opportunity to move at their own pace and choose what level of difficulty they wanted to begin at. When I expressed to the entire class that students had solved the hard activities correctly it motivated all of the groups to work harder because they knew it was an attainable goal.

Confidence was a trait that not all of my students possessed about math according to the surveys they had completed at the beginning of the study. Thirty nine percent of my students indicated that they did not feel confident and forty

four percent selected that math scared them. Unfortunately, these results did not change significantly at the end of the study according to my post-survey. I know that when students master specific skills, teachers expect that the students can go further, so they continue to increase their level of difficulty in the content areas. My students may continue to feel scared and nervous because the expectations continue to rise throughout the school year so that we see increased mathematical understanding.

The students completed a pre-survey and post-survey to measure their attitudes and perceptions about math and how confident they felt, if they enjoy the subject, if they enjoy challenge, and they think math is fun. Table 1 below explains the data from the beginning of the study compared to the conclusion using the questions that showed the most change from pre-study to post-study. A complete table that includes the 17 questions is located in Appendix K.

Table 1: Pre-survey and post-survey results from questions that revealed the most change.

<u>Survey Question</u>	<u>Pre-Survey</u>	<u>Post-Survey</u>
I enjoy math.	37% Always	56% Always
Math scares me.	61% Never	78% Never
I think math is boring.	67% Never	78% Never
I would like to be smarter in math.	78% Always	83% Always
I pay attention in math.	72% Always	78% Always

Motivation is almost always included when students are working with peers in cooperative learning groups rather than independently. McCrone (2005) and Manouchehri & Enderson (1999) supported the notion of group work increases motivation as well as develops students' communication through the use of dialogue and finding ways for the students to defend and/or justify their answers to group members.

- *I have evidence that drawing pictures and engaging in dialogue when solving mathematical word problems helped my students to improve their depth of mathematical understanding.*

Drawing visual representations of what the students comprehend during the reading of a word problem assists students in understanding the problem. Martiniello (2007) and Abedi and Lord (2001) support the drawing of visual representations as well. They also agree that when students have a picture, they can approach the problem solving process because a picture helps to clarify the problem. A study conducted by DeCorte, Verschaffel, and Masui (2004) found the addition of a mental representation led to successful problem solving among the students as well.

The cow and duck problem was where my students were expected to draw a picture to assist them in solving the problem. Once they understood that they had to know that cows had four legs and ducks had only two, they could begin their pictures using how many heads were given in the problem. I continued to provide them with problems throughout the course of the study that required them to draw pictures, write mathematical number sentences and explain in words what they did to solve the problem. Giving my students multiple opportunities to practice using the strategy of drawing pictures, using numbers, and writing words allowed me to extend their learning to creating number sentences to describe what they completed mathematically in the word problem.

These activities continued to reinforce student learning and allowed the students to practice all skills that they have been learning in problem solving context. The addition of using dialogue assisted all students through the process of solving a problem. These student work activities throughout the study pushed my students to develop their understanding of more challenging problems. As student work was gathered and examined, the majority of my students were continuing to improve their understanding of word problems.

Although the visual representations helped my students understand the problem, the move to the higher cognitive level in the written format was a challenge for most of my students. As evidenced from my student interviews, many of my students did not enjoy the written portion of the activities. This step

in solving word problems was the most cognitively demanding task of the entire study that was required for all my students. In observational notes, students were able to justify to their classmates correct and incorrect solutions and why, but the step to explaining how they solved the problem frustrated a lot of my students. I continued to reinforce to my students that they should draw pictures and write number sentences and leave the words section for last so that they can use their pictures and numbers to help guide them in their written explanations.

The “making change” pre-test in words was used as a baseline measure of student growth throughout the study (Appendix F). My students needed to be able to add \$10 and three \$1 bills together and then subtract \$12.62 from the total to find the difference. They also needed to give two different ways to show the change and describe which way used the fewest coins. The results from this assessment is as follows: 50% of my students did not answer the problem correctly and therefore scored a zero, 17% scored a one, 17% received a two, and the remaining 16% scored four in their written explanations. The half of my students that scored a zero had answers where they just added the money together, forgot to add the \$3 to the \$10 for the total amount, or their answer did not match what the problem was asking. The students that received a two or a one because they added the three \$1 bills together with the \$10 bill, but then didn’t subtract correctly or had another error.

Table 2: The results of the pre-test in words.

Student Scores (range from 0-4; 4 being highest)	Number of Students
0	9
1	3
2	3
3	0
4	3

At the conclusion of my study, I conducted an individual post-test in words to measure student growth. The content was the same; adding two amounts of money and then subtracting the total spent from an amount. The table below shows the results of the post-test in words.

Table 3: Results of the post-test in words.

Student Scores (range from 0-4; 4 being highest)	Number of Students	Percentage of Improved Change from Pre to Post-test
0	0	50%
1	0	17%
2	4	5%
3	5	28%
4	9	33%

In addition to the students writing an explanation how they solved the problem, the students also completed a visual representation and wrote number sentences showing what they did mathematically to solve the problem. The growth in the student's understanding was evident by these scores. On the post-test, 0% of my students scored either a 0 or 1 (not scored or below basic), 22% received a 2 (basic score), 28% received a 3 (proficient), and 50% scored a 4 (advanced) on the assessment.

Another assessment that showed students depth of mathematical understanding was the benchmarking assessment. The table below shows individual student growth on the Study Island Benchmarking Assessment for the math standards in third grade. As shown in the third column, 11 students improved one or more proficiency levels, seven remained at the same level, and 0 students regressed. This assessment was given to all students as a means to gather data about mathematical understanding of my students at the beginning of the school year. Data included questions about students' number sense and operations, measurement, geometry, algebraic concepts, and data analysis and probability. This assessment has a high correlation with the PSSAs that the students take later on in the school year; therefore, how the students perform on the Study Island assessment is a predictor of how they will perform on the state mandated tests. An advanced scaled score ranges from high 1827 to low 1370, a

proficient score ranges from high 1369 to low as 1180, a high basic score is 1179 to low as 1044, and a below basic score ranges from high 1043 to low as 750.

Table 4: Study Island scaled scores from beginning of year to midyear.

Student Name	Benchmark Scaled Score Study Island	Mid-Year Scaled Score Study Island	Standards Proficiency Level
Jenga	1118	1609	Basic to Advanced
Adrian	1609	1609	Advanced- no change
Drake	1344	1325	Proficient- no change
May	1028	1301	Below Basic to Proficient
Taylor	1118	1301	Basic to Proficient
Jessica	1152	1718	Basic to Advanced
Ginka	1312	1350	Proficient- no change
Christine	1501	1827	Advanced- no change
Max	1281	1350	Proficient- no change
Ella	973	1325	Below Basic to Proficient
Thomas	1281	1501	Proficient to Advanced
June	1392	1392	Advanced- no change
Janice	1051	1277	Basic to Proficient
Jose	1010	1277	Below Basic to Proficient
Steven	1501	1609	Advanced- no change
Jean-Marie	1118	1392	Basic to Proficient
Isabel	973	1277	Below Basic to Proficient
Lina	1028	1501	Below Basic to Advanced

The results of Johnstone (2006), Esmonde (2009), and Weber (2008) were consistent with the results of my study in that cooperative learning groups increased student understanding and lead each of the group members through the process of problem solving. Because my students were engaged in problem solving dialogue, each student contributed to other students' understanding of the problem solving activities. The students were explaining and justifying their own answers and each member of the groups agreed with or disagreed with one another on the answers given.

Results from my pre-test in Study Island and the post-test as well, indicated improved student mathematical learning across the topics (Table 4). Also, the students improved in single and multiple step word problems as seen in Table 6.

- *I have evidence that encouraging dialogue between students in cooperative learning groups when solving mathematical word problems helped my students to strengthen their higher level logic, reasoning skills, and number sense.*

The NCTM recommends the use of dialogue in small groups to facilitate mathematical understanding among the individual students. The interaction that the students provide one another in the group allows the students to remain engaged in the material and aiding one another through the understanding of the

text allowing the problem solving to occur. NCTM notes that when students are engaged either in cooperative learning groups or in a whole group environment, the fact that students are evaluating others responses of mathematical thinking pushes all involved to higher level thinking. No longer are students just solving problems, they are evaluating and synthesizing information. Weber et al. (2008) found that when students are engaged in dialogue they are providing justification for their mathematical thinking, which is improving their higher-level thinking. Vygotsky (1978) believed that when students are engaged in collaborative situations using dialogue and communication, the zone of proximal development is growing as a result of the interaction.

The design and methodology of this study used cooperative learning groups in each of the assignments, except in testing situations when it was individual. Students were taught that each student was accountable for how she or he participated in collaborative settings, being engaged, and on task while their group was working. The documentation I have in my observational field log provides evidence that all of my students were engaged in mathematical dialogue during problem solving class each day. The matrix problems, all problem solving performance assessments, as well as the brainteasers targeted skills surrounding the mathematical areas of logic, reasoning and number sense. Specifically, the change range, fruit salad, and trip chips logic problems were on a fourth grade

level. All student work collected served as documentation that students were completing assignments and focusing on the work that was assigned.

The table below shows the growth of the number of students at each level of proficiency comparing the benchmark levels to the mid study levels of students' mathematical understanding. At the beginning of the year, 10 students were below proficiency on the assessment, but by midyear, which was also the mid point of my study, those 10 students moved into proficiency or the advanced level on the same assessment.

Table 5: Benchmark testing of students comparing beginning of year math to middle of the year math.

Level of Proficiency	Study Island Benchmark 1 Number of Students	Study Island Benchmark 2 Number of Students
Below Basic	5	0
Basic	5	0
Proficient	4	9
Advanced	4	9

- *I have evidence that encouraging dialogue between students in cooperative learning groups when solving mathematical word problems helped my students to improve their written and verbal skills.*

Using the data collected from the post-test in words (Appendix L), I was able to observe that most students were able to independently complete a multiple step word problem and then explain in numbers and words how they solved the problem. One example of student work is included below.

<p style="text-align: center;">Writing Prompt:</p> <p>Joanna bought a soda for \$1.83. She also bought a pack of gum for \$1.96. She paid the clerk with a \$5 bill. How much money did Joanna get back in change? Solve using pictures, numbers, and words. Explain how you solved the problem in your words section.</p> <p style="text-align: center;">Writing Composition (original):</p> <p>First, I added <math>\\$1.83 + \\$1.96</math> and I got <math>\\$3.79</math>. Next, I knew that she paid with a <math>\\$5.00</math> bill. Then, I subtracted <math>\\$5.00 - \\$3.79</math> and I got <math>\\$1.21</math>. Finally, I know that she got <math>\\$1.21</math> in change.</p> <p style="text-align: center;"><math>\\$1.83 + \\$1.96 = \\$3.79</math>    <math>\\$5.00 - \\$3.79 = \\$1.21</math>.</p>
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*Figure 25: shows individual student work for the post-test involving pictures, numbers, and words*

The following table compares the students' written explanations on the steps they took to solve the multiple step word problems in the beginning of the study (Appendix D) versus their written explanations at the end of the study (Appendix L). I kept the content of the word problem consistent as well as the skills. The problem solving rubric was used to score the open ended responses as well as the picture and number section on both assessments (Appendix J). Twelve of my students improved their scores from the beginning of the study to the end 2

or more points of proficiency, zero students decreased in their scores, and six either stayed the same or improved three or more levels of proficiency.

Table 6: Compares students' open-ended responses to multiple step word problems.

Name	Written Explanation Score out of 4 (Advanced) Pre Test	Study Island Written Explanation Score out of 4 (Advanced) Post Test
Jenga	0	2
Adrian	4	4
Drake	1	2
May	0	4
Taylor	0	2
Jessica	0	4
Ginka	0	4
Christine	2	4
Max	2	3
Ella	0	3
Thomas	4	4
June	4	4
Janice	0	3
Jose	1	3
Steven	2	4
Jean-Marie	0	3
Isabel	1	2
Lina	0	4

The table below summarizes the students' scores on the pre-test written explanations compared to the post-test explanations. The pre and post test questions and rubric are given in Appendix D and J. 17% of my students remained at the advanced level, 17% increased one level of their proficiency, 28% moved two levels, 17% increased three levels, and 22% increased four levels on their written responses to a multiple step word problem.

Table 7: Student scores comparing pre-test in words to post-test in words.

Student Scores (range from 0-4; 4 being highest)	Number of Students Pre-Test in Words	Number of Students Post-Test in Words
0	9	0
1	3	0
2	3	4
3	0	5
4	3	9

Throughout my observational journal, I documented student dialogue on a consistent basis covering all areas of number sense, logical and reasoning skills, higher level thinking, and general connections of mathematical skills that my students were making. Beginning at the start of my study, I introduced problem solving and dialogue through Sudoku puzzles. The following activities were given in a sequence that would gradually increase the dialogue between students

because the level of difficulty continued to rise. Most of the groups, unfortunately not all, continued to discuss and verbally support each other in solving the mathematical word problems and other problem solving activities. The groups that were not working collaboratively received support from the special education teacher and myself and we helped to facilitate the dialogue. Observations noted that the struggling math students did not know where to begin when solving the problems, or they started somewhere with incorrect answers and did not know where to go from there. We supported them by asking them to tell us how they began and why. From there, we guided them through the discussion of what step they should take next to point them in the right direction.

Kieren (2001) found that as long as students were expected to provide written responses after the completion of problem solving activities, their mathematical understanding improved. The study had students working in pairs only after they had completed the problem independently. Afterwards, the pairs of students would engage in dialogue to explain how they solved the problem and the study found that the discourse helped the understanding of both partners involved. Zack and Graves (2001) conducted a similar study compared to Kieren. Zack and Graves found that the dialogue and understanding depended on the levels of understanding that were produced in the groups. So, if students were working in homogenous groups, but the level of that group included mostly struggling students, the dialogue would not push the others to higher cognitive

understanding. If the groups included some academically stronger students, the levels of all of the students in that group would improve.

## Next Steps

The following three quotes were chosen as the most representative of what guided my research study.

“[Students] can’t survive without good teachers and, no matter what curriculum may be in place, whether it’s approved by state officials or by Washington or not, they are not good at all if teachers are unable to enjoy the work they do and be invigorated by its unpredictables” (Kozol, 2005, p. 299-300).

This quote encapsulates my study. Coding my field log allowed me to see how much fun the students and I were having throughout the course of the study. I felt that I had the freedom to incorporate materials that I had found with materials that were required of the curriculum and use them to continue to motivate my students to complete complex tasks through cooperative learning. I had the opportunity to be energized by my students and needed to be ready for anything that happened throughout my study. When I observed my students and could sense their frustration, I continued to only give them specific support so that they would continue the problem solving in their groups. Students feel the emotion that their teachers emit and emotions of stress or negativity affect attitudes and learning.

I created activities that I thought were just above the majority of my students' mathematical levels, listened and observed the groups every day in order to gauge if the activities were too challenging or appropriate.

“The teacher is no longer merely the-one-who-teaches, but one who is himself taught in dialogue with the students, who in turn while being taught also teaches. They become jointly responsible for a process in which all grow” (Freire, 1970, p. 80).

The teacher is not the only being that possesses power in which to teach others, the students can also teach the teacher. Both teachers and students become responsible for teaching one another through the use of dialogue and communication. Dewey (1938) speaks of the importance of the quality of activities and the experiences, which the teacher incorporates into the learning. If teachers provide experiences with little planning, implementation, or without a sufficient amount of directive and management, the experiences will not necessarily lead to successful learning. Experiences should not occur just to happen, there should be careful consideration of the process, product, and other areas of the educational process that need justifiable and age appropriate activities so that all students learn the objectives.

The students were working within their zone of proximal development in groups and it allowed them to discuss and participate with one another.

“Learning is more than the acquisition of the ability to think; it is the acquisition of many specialized abilities for thinking about a variety of things. Learning does not alter our overall ability to focus attention but rather develops various abilities to focus attention on a variety of things.” (Vygotsky, 1978, p. 83).

Vygotsky is speaking of the fact that education should not be so focused that students only learn the narrowest concepts in a few areas. Education should have depth and be generalized in many diverse areas. Students need to understand not only the superficial concepts and skills, but also how the different disciplines are intertwined and related to one another. Learning is about developing our ability to focus, not in one area, but in all areas so we develop all of our talents in a wide variety of specialties.

I believe that students should be taught across all content areas, but on a deeper level. Education should not just skim across the top of the curriculum, just focusing on student understanding on a basic application level. Students should be pushed to higher cognitive levels, such as synthesis and evaluation in order to develop full understanding. I focused on dialogue and discussion, which is a higher cognitive level compared to application level of the skills. I used problem

solving to teach logic and reasoning, number sense, higher-level thinking, pictorial representations, and open-ended responses to mathematics problems.

I will continue to develop activities that support higher-level thinking in mathematics as well as develop the communication skills of my students. I want to deepen and strengthen students' mathematical understanding so that they understand the "big picture" and form a solid foundation for future learning. As a teacher, my job is to instill a life long passion for learning as well as ignite a flame that is infectious among all learners.

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## Appendix A: HSIRB Approval Letter



June 8, 2011

Jennifer Eisenhard  
309 Hill Road  
Whitehall, PA 18052

Re: HSIRB proposal by Jennifer Eisenhard for Richard Grove

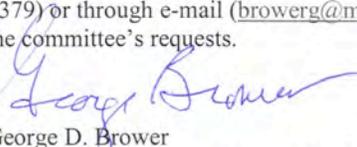
Dear Jennifer Eisenhard:

The Moravian College Human Subjects Internal Review Board has accepted your proposal: "Discussion and Dialogue in a 3rd Grade Math Problem Solving Class." Given the materials submitted, your proposal received an expedited review. A copy of your proposal will remain with the HSIRB Chair.

Please note that if you intend on venturing into other topics than the ones indicated in your proposal, you must inform the HSIRB about what those topics will be.

Should any other aspect of your research change or extend past one year of the date of this letter, you must file those changes or extensions with the HSIRB before implementation.

This letter has been sent to you through U.S. Mail and e-mail. Please do not hesitate to contact me by telephone (610-861-1379) or through e-mail ([browerg@moravian.edu](mailto:browerg@moravian.edu)) should you have any questions about the committee's requests.



George D. Brower  
Chair, Human Subjects Internal Review Board  
Moravian College  
610-861-1379

## Appendix B: Parent/Guardian Consent Form

October 20, 2011

Dear Parents/Guardians,

I am currently working to achieve my Master's Degree in Curriculum and Instruction from Moravian College. One of the course requirements of *Curriculum Development and Action Research* is the completion of an action project. The question I will be targeting is the addition of scoring student discussion patterns in my third grade problem solving math class.

The purpose of the study is to see how modeling appropriate questioning techniques and discussion for my students in mathematics-based problem solving activities will strengthen problem solving skills on PSSA's and also improve student attitude towards solving word problems. A detailed scoring rubric identifying multiple levels of understanding will show me exactly where students are in their understanding. My students will continue to work in small groups, pairs, and triads during this time. They will complete a survey at the beginning of the study asking for their feelings towards math, problem solving, group work, etc. Students will complete the same survey at the project's end in order to see growth and improvement. The goal of my study is to improve student/teacher and student/student discourse while increasing their understanding of math concepts. I will be using data from a variety of areas all within School District's requirements. All students' names will be kept confidential. All students will take part in the mathematics problem-solving activities as part of the regular curriculum but I will use only the data from research participants in the study. Please note that any of my students may withdraw from this study at any time without penalty.

Should you have any questions regarding the study, please feel free to contact me at \_\_\_\_\_ or [jeisenhard@bethsd.org](mailto:jeisenhard@bethsd.org). My faculty sponsor at Moravian College is Dr. Joseph Shosh and he can be reached at 610-861-1482 or by email at [jshosh@moravian.edu](mailto:jshosh@moravian.edu). Please sign and return the bottom portion of the form at your earliest convenience. I thank you very much for your support.

Sincerely,

Miss Jennifer Eisenhard, 3<sup>rd</sup> grade teacher

\_\_\_\_\_ I will allow Miss Eisenhard to use the information gathered from the study at \_\_\_\_\_ School.

\_\_\_\_\_ I do not allow Miss Eisenhard to use the information gathered from the study at this time.

Parent/Guardian's Signature \_\_\_\_\_ Date: \_\_\_\_\_

Child's Name \_\_\_\_\_

## Appendix C: Principal Consent Form

September 20, 2011

Dear

I am currently working to achieve my Master's Degree in Curriculum and Instruction from Moravian College. In accordance with the course requirements of *Reflective Practice Seminar*, completion of an action project, which is narrowly focused with measurable results, is mandated. The question I will be targeting is the implementation of accountability in student discourse patterns in my third grade problem solving math class.

The purpose of the study is to see how modeling appropriate questioning techniques and discussion for my students in mathematics-based problem solving activities will strengthen problem solving skills on standardized tests and also improves student attitude regarding word problems. A detailed rubric identifying attributes of multiple levels of competency will ensure accountability. My students will continue to work in small groups, pairs, and triads optimizing their discourse opportunities during this time. They will complete a survey at the beginning of the study asking for their feelings towards math, problem solving, group work, etc. In an effort to maintain high levels of validity, students will complete the same survey at the project's end. The goal of my study is to improve student/teacher and student/student discourse while increasing their understanding of math concepts. I will be using data from Study Island, performance assessments, problem solving activities from the Scott Foresman math series, and also a large collection of observational data. All students and their work will be identified through the use of pseudonyms. All students will take part in the mathematics problem-solving activities as part of the regular curriculum, but I will use only the data from research participants in the study. Please note that any of my students may withdraw from this study at any time without penalty.

Should you have any questions regarding the study, please feel free to contact me. My faculty sponsor at Moravian College is Dr. Joseph Shosh and he can be reached at 610-861-1482 or by email at [jshosh@moravian.edu](mailto:jshosh@moravian.edu). I am asking that you sign and date the form below and return it as soon as possible. I thank you very much for your support.

Sincerely,

Jennifer Eisenhard

I attest that I am the principal of the teacher conducting this research study and that I have read and understand the consent form. Ms. Jennifer Eisenhard has my permission to conduct this study at \_\_\_\_\_ School.

Principal's Signature



Date: 9/21/11

## Appendix D: Problem Solving Comparing Numbers

Name \_\_\_\_\_

PS 1-7

### Comparing Numbers

**Middletown** Use the chart below for Exercises 1–3.

Facts About Middletown	
Number of people .....	5,867
Students in grade school .....	1,010
Students in high school .....	732
Number of houses .....	2,100
Books in the library .....	8,463

1. Compare the number of people in Middletown to the number of books in their library. Use  $<$ ,  $>$ , or  $=$ .

number of people \_\_\_\_\_ number of books

2. Compare the number of students in grade school to the number of students in high school.

number of students in grade school \_\_\_\_\_ number of students in high school

3. Write a sentence to compare the number of houses to the number of students in grade school. Use the words *greater than* or *less than*.

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4. **Writing in Math** Which is greater, thirteen hundred or one thousand, two hundred? Explain how you solved this problem.

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## Appendix E: Problem Solving Counting Money

Name \_\_\_\_\_

### Counting Money

PS 1-12

**School Lunch** There are two choices for lunch at Lincoln Elementary School. Lunch A costs \$3.50 and Lunch B costs \$3.35.

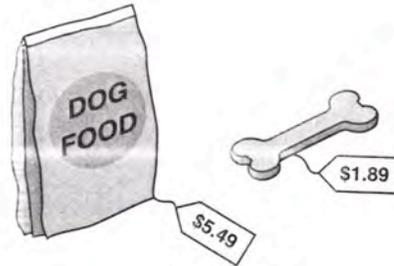
1. What bills and coins could be used to pay for Lunch A?

\_\_\_\_\_

2. What bills and coins could be used to pay for Lunch B?

\_\_\_\_\_

**Pet Supplies** Molly needs to buy some supplies for her new puppy. At the pet store, she will buy some of the things she needs.



3. What bills and coins could Molly use to pay for the dog food?

\_\_\_\_\_

4. Molly has 6 quarters, 2 dimes, and a nickel. Does she have enough to buy the dog bone? Explain.

\_\_\_\_\_

\_\_\_\_\_

5. **Writing in Math** Tell what coins you could use to make \$0.75 in three different ways.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

## Appendix F: Problem Solving Making Change

Name \_\_\_\_\_

### Making Change

PS 1-13

**Bookstore** For 1–3 use the chart below. List the coins and bills used to make change. Then write the change in dollars and cents.

Bookstore Prices	
Paperback books .....	\$4.75
Hardcover books .....	\$8.89
Magazines .....	\$2.58

1. Lauren bought a magazine. She paid with three \$1.00 bills.

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2. Theresa bought a hardcover book. She paid with a \$10.00 bill.

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3. Maria bought a paperback book. She paid with a \$20.00 bill.

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4. **Writing in Math** John went to the garden store and bought a shovel that cost \$12.62. He paid with a \$10.00 bill and three \$1.00 bills. Give two ways to make the change. Which used the fewest coins?

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## Appendix G: Pre- Study and Post- Study Survey

### Pre-Study/Post-Study Self Evaluation in Math

Name: \_\_\_\_\_ Date: \_\_\_\_\_

I enjoy math.	Never	Seldom	Sometimes	Usually	Always
I think I am good at math.	Never	Seldom	Sometimes	Usually	Always
I try to participate a lot in math class.	Never	Seldom	Sometimes	Usually	Always
I feel confident in math class.	Never	Seldom	Sometimes	Usually	Always
Math scares me.	Never	Seldom	Sometimes	Usually	Always
I feel nervous in math class.	Never	Seldom	Sometimes	Usually	Always
I think math is boring.	Never	Seldom	Sometimes	Usually	Always
I feel pressured in math class.	Never	Seldom	Sometimes	Usually	Always
I would describe math as fun.	Never	Seldom	Sometimes	Usually	Always
I would like to be smarter in math.	Never	Seldom	Sometimes	Usually	Always
I like math when it is challenging.	Never	Seldom	Sometimes	Usually	Always
I like math when it is easy.	Never	Seldom	Sometimes	Usually	Always
When I think about math, I think I can learn everything taught.	Never	Seldom	Sometimes	Usually	Always
I like to solve challenging problems.	Never	Seldom	Sometimes	Usually	Always
I pay attention during math.	Never	Seldom	Sometimes	Usually	Always
I enjoy math when the lesson includes a hands-on activity.	Never	Seldom	Sometimes	Usually	Always
I think math is important because it will help me in the future.	Never	Seldom	Sometimes	Usually	Always

Adapted from DeSanctis. 2009.

## Appendix H: Student Interview Questions

Student Number: \_\_\_\_\_

Date: \_\_\_\_\_

1. What do you like most about math and why?

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2. What do you least like about math and why?

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3. What do you like and dislike about word problems?

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4. What are you learning? How can you use your new learning?

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## Appendix I: Study Island Post Test

### Question #1

Three friends went to the batting cage. Cameron made 31 hits, Austin made 16 hits, and Jordan made 24 hits. How many hits did the three make in all?

- A. 81
- B. 71
- C. 40
- D. 47

### Question #2

Michelle works at a kitchen making sandwiches. She has made 26 sandwiches so far today and has sold 13 of them. If she makes 19 more, how many sandwiches will she have for sale?

- A. 13
- B. 32
- C. 58
- D. 45

### Question #3

There were 41 students on the kindergarten playground and 32 students on the second-grade playground at Robertson Elementary. How many students were on the two playgrounds in all?

- A. 63
- B. 74
- C. 73
- D. 75

### Question #4

Hunter and Logan built small sand towers on the beach. Hunter built 20 towers, and Logan built 8 towers. Then, the ocean water came up and knocked down 5 of the towers. How many of the boys' sand towers were left standing?

- A. 7
- B. 27
- C. 23
- D. 17

## Appendix I: Study Island Post Test Continued

### Question #5

After lunch, Caleb's mom said he had to wait 35 minutes before he could swim. Caleb has already waited 24 minutes. How many more minutes does Caleb still need to wait before he can swim?

- A. 59
- B. 10
- C. 11
- D. 13

### Question #6

After lunch, Caleb's mom said he had to wait 45 minutes before he could swim. Caleb has already waited 39 minutes. How many more minutes does Caleb still need to wait before he can swim?

- A. 3
- B. 84
- C. 10
- D. 6

### Question #7

Sydney and Grace brought 16 slices of bread with them to the park. They broke the slices into tiny pieces and fed them to the ducks. Sydney fed the ducks 3 slices, and Grace fed them 10 slices. How many slices of bread did they have left?

- A. 3
- B. 6
- C. 1
- D. 13

### Question #8

Hunter and Logan built small sand towers on the beach. Hunter built 18 towers, and Logan built 12 towers. Then, the ocean water came up and knocked down 6 of the towers. How many of the boys' sand towers were left standing?

- A. 12
- B. 26
- C. 24
- D. 0

### Appendix I: Study Island Post Test Continued

#### Question #9

Michelle works at a kitchen making sandwiches. She has made 30 sandwiches so far today and has sold 10 of them. If she makes 17 more, how many sandwiches will she have for sale?

- A. 23
- B. 37
- C. 20
- D. 47

#### Question #10

Hunter and Logan built small sand towers on the beach. Hunter built 19 towers, and Logan built 11 towers. Then, the ocean water came up and knocked down 4 of the towers. How many of the boys' sand towers were left standing?

- A. 34
- B. 4
- C. 27
- D. 26
- E. 12

#### Question #11

Sydney and Grace brought 17 slices of bread with them to the park. They broke the slices into tiny pieces and fed them to the ducks. Sydney fed the ducks 5 slices, and Grace fed them 7 slices. How many slices of bread did they have left?

- A. 10
- B. 1
- C. 12
- D. 5

#### Question #12

Sydney and Grace brought 19 slices of bread with them to the park. They broke the slices into tiny pieces and fed them to the ducks. Sydney fed the ducks 6 slices, and Grace fed them 9 slices. How many slices of bread did they have left?

- A. 4
- B. 15
- C. 10
- D. 13

## Appendix J: Scoring Rubric for Problem Solving Activities

Student Number: \_\_\_\_\_

Date: \_\_\_\_\_

Criteria	Beyond Standards 4 points	Meets Standards 3 points	Not Yet Meeting Standards 2 or 1 point	Score
Pictures accurately match number sentence	Pictures match number sentence and are labeled	Pictures match number sentence	Pictures not included or are incorrect	
Accurately written number sentence	Number sentence is accurate and may also include another type of number sentence (3 or 4 examples)	Number sentence accurately shown and answered correctly (3 examples correct)	Missing or inaccurate number sentence (2 or fewer examples)	
Words section show understanding of mathematical process in solving mathematical problems	All ideas clearly written explain how multiplication is understood (possibly through other real world examples-generalization)	All ideas are written explain process of understanding multiplication	Missing, incomplete, or incorrect written understanding of multiplication	
Words section explains key steps involved in solving the problem(s)	All steps correctly included in written explanation	Some of the steps are included in written explanation	Missing, incomplete, or incorrect steps used in the written explanation	

\* A score of 0 indicates the student did not attempt the problem or the answer included a response that had no connection with the problem

### Appendix K: Pre and Post Survey Results

Question	Pre-Survey	Post-Study
I enjoy math.	72% Always/Usually	94% Always/Usually
I think I am good at math.	67% Always/Usually	72% Always/Usually
I try to participate a lot in math class	83% Always/Usually	78% Always/Usually
I feel confident in math class.	94% Always/Usually	72% Always/Usually
Math scares me.	67% Never/Seldom	83% Never/Seldom
I feel nervous in math class.	67% Never/Seldom	78% Never/Seldom
I think math is boring.	83% Never/Seldom	100% Never/Seldom
I feel pressured in math class.	44% Never/Seldom	67% Never/Seldom
I would describe math as fun.	78% Always/Usually	78% Always/Usually
I would like to be smarter in math.	100% Always/Usually	83% Always/Usually
I like math when it is challenging.	72% Always/Usually	56% Always/Usually
I like math when it is easy.	39% Never/Seldom	39% Never/Seldom
When I think about math, I think I can learn everything taught.	67% Always/Usually	61% Always/Usually
I like to solve challenging problems.	94% Always/Usually	56% Always/Usually
I pay attention during math.	89% Always/Usually	89% Always/Usually
I enjoy math when the lesson includes a hands-on activity.	100% Always/Usually	83% Always/Usually
I think math is important because it will help me in the future.	100% Always/Usually	100% Always/Usually

### **Appendix L: Post Study In Words**

Joanna bought a soda for \$1.83. She also bought a pack of gum for \$1.96. She paid the clerk with a \$5 bill. How much money did Joanna get back in change? Solve using pictures, numbers, and words. Explain how you solved the problem in your words section.