

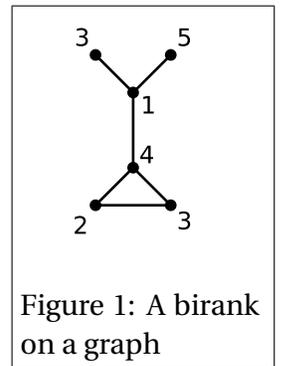
SOAR Research Proposal – Summer, 2012

Graph Biranking

Title: Graph Biranking
Faculty: Michael Fraboni, Associate Professor, Dept of Mathematics and Computer Science
Student: Madison Zebrine
Timeline: 10 weeks beginning May 29, 2012

Description of the project

We propose to work on a problem in the field of Graph Theory. A graph is a collection of points, called vertices, and edges connecting some of those points. A *birank* on a graph is an assignment of a whole number, or rank, to each vertex so that whenever two vertices have the same number, any path connecting those vertices contains a number which is larger and a number which is smaller. For example, note in Figure 1 there are two vertices with an assigned rank of 3. When following any path between these two vertices we find a vertex with rank 1 and a vertex with rank 4. Typically the goal is to find a biranking which uses the smallest numbers possible. For instance, Figure 1 does not represent the most efficient biranking since the 5 could be replaced with a 2 and we would still have a valid biranking.



In a recent paper my coauthors and I found the minimum number of labels needed to create a birank for some specific families of graphs. I would like to work with Madison to expand on these results. Specifically, can we compute the minimum number of labels needed to create a birank for other classes of graphs? Do the techniques in my paper generalize or are new ones required?

The notion of a biranking on a graph is a relatively new idea and is seeing interest in the field. In addition to this being an interesting problem in its own right, this is an area of graph theory that has applications in problems such as optimizing layouts of components on an integrated circuit.

Roles and Responsibilities

Madison is a talented and curious young mathematician who will take an active part in this research project. She will begin with a literature survey and background reading. Most likely she will start with my recent paper and its bibliography and branch out from there as necessary. I will help her to understand the ideas and approaches in these papers as necessary. Once she has a sufficient grasp of the problem and its context we will begin to experiment with various graph families, look for patterns, and make conjectures based on these observations. After we have identified some promising conjectures, Madison and I will develop formal proofs and generalize where appropriate. I anticipate this project will result in a publishable paper, so in the final weeks of our project we will organize our work into a paper suitable for submission to a research journal.

Below is an approximate time line for this project. There will likely be some overlap between these categories. For instance, some writing will probably occur throughout the project, but I expect the majority to occur near the end.

- 2 weeks: Background reading
- 2 weeks: Experiment with several graph families, look for patterns, and develop appropriate conjectures
- 4 weeks: Prove conjectures and seek to generalize as appropriate
- 2 weeks: Writing

Student Involvement

This project will give Madison a taste of the excitement of working at the edge of mathematical knowledge. She will gain an understanding of what it means to do research in mathematics: work to understand writings from different authors, identify important questions, then go beyond what is written to conjecture and prove new theorems.

As detailed above, Madison will be involved in every aspect of the project. She will work with me to decide what are the best questions to ask, to develop conjectures based on experimentation, and to prove her conjectures. I anticipate she will do most of the writing, though I expect to provide significant input as she learns how mathematical research papers are structured.

Student Contribution to the Discipline

The work we do this summer should help to answer interesting open questions in the field of graph theory. Madison will be contributing directly to the discipline's discussion of the biranking of graphs.

Madison will have the opportunity to share her work in several venues. First, we intend to have a paper appropriate for submission to a research journal. Next, we plan for her to present aspects her work at the regional meeting of the Mathematical Association of America in the fall, as well as at the Moravian College Undergraduate Mathematics conference next winter. Throughout the academic year the student Math Society hosts short mathematical talks which will be the perfect venue for Madison to share her work with other students at Moravian. Finally, of course, she will participate in Scholar's day next spring – another chance to share her work with her Moravian colleagues.

Budget items

We anticipate no equipment, supplies, or travel expenses.

Graph Biranking

Madison Zebrine, Mathematics and Philosophy, May 2013

Dr. Michael Fraboni

On-campus housing is requested

Doing a SOAR project this summer is an excellent opportunity to expand my knowledge of and appreciation for mathematics I am considering attending graduate school for mathematics and participating in this SOAR project will give me the opportunity to learn what math research is and if it's something that I would like to spend the next few years - and beyond - of my life doing. With the exception of a short paper for Discrete Math, I've never done math research, and I know very little about the process. If I do decide to attend graduate school in mathematics, participating in SOAR will give me a greater understanding of how to conduct research. I also am planning on doing an Honors project next year, and a SOAR project will give me prior experience in math research before diving in a research project during the school year when I have other obligations and only the slightest idea of what I'm doing.

Participating in this project will give me the opportunity to enter the world of mathematical research. By reading recent papers published in the field and learning how to ask my own questions based on the previous research and working on figuring out answers, tweaking the questions when necessary, my understanding of and appreciation for mathematics will grow. Graph theory is a growing area of mathematics, and biranking of graphs is an even newer area of graph theory, so I have the chance to contribute new, significant knowledge to the field. I will also have the opportunity to present my research at conferences, where I can meet other mathematicians and continue to expand my knowledge of math.

After participating in this SOAR project, I will understand the entire process of conducting mathematical research. I will learn how to refine the questions for exploration I ask myself as I read math papers and expand my critical thinking skills and creativity as I work out possible answers to the questions. At the end of the project, I expect to have a paper suitable for publication and presentation at various conferences, which will give me the opportunity to practice writing polished math proofs and papers explaining my research. By participating in SOAR, I will grow as a mathematician.